MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.5 (B-1.8)

(Computational Fluid Dynamics - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Use a separate Answer-Script for each part.

(Notations have their usual meanings.)

Part - I

(25 Marks)

Answer Question No. 1 and any *two* from the rest.

1. Explain Crank-Nicolson implicit finite difference scheme for solving one-dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}; \quad a \le x \le b, \quad t > 0$$

[*Turn over*]

subject to the following initial and boundary conditions :

$$u(x,\ 0) = f(x); \quad a < x < b$$

$$u(a,\ t) = \phi(t), \quad u(b,\ t) = \psi(t); \quad t > 0 .$$

If the derivative boundary conditions

$\frac{\partial u}{\partial x} - u = 0$ at $x = a$ and $\frac{\partial u}{\partial x} + u = 0$ at $x = b$ are provided then give the computational scheme for u at the boundaries. 6+3

2. Establish alternating direction implicit (ADI) method for the solution of partial differential equation

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \text{ where } k = \text{constant} > 0.$$

Also give a physical sketch of the grid generation of ADI scheme. 6+2

3. Deduce explicit finite difference scheme for the solution of second order wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \ ; \ 0 \le x \le 1,\ t > 0$$

[*Turn over*]

subject to the following initial and boundary conditions :

$$u(x,\ 0) = f(x),\ \left[\frac{\partial u}{\partial t}\right]_{(x,\ 0)} = g(x);\quad 0 < x < 1$$

$$u(0,\ t) = \phi(t),\ \ u(1,\ t) = \psi(t);\quad t > 0.$$

Hence investigate the von-Neumann stability analysis. 4+4

4. Derive finite difference scheme for solving 2-D Poisson's equation of the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,\ y) \text{ in } \Omega : (x,\ y) \in [0,\ 1] \times [0,\ 1]$$

subject to the homogeneous Dirichlet boundary condition $u = 0$ on $\partial\Omega$.

Hence represent the finite difference scheme in block diagonal or sparse matrix form. Also state the iterative procedures for solving above finite difference scheme.

5+1½+1½

[*Turn over*]

Part - II

(25 Marks)

Answer Question No. 5 and any *one* from the rest.

5. Give an elaborate account of solving one dimensional unsteady convection diffusion problem using finite volume.

   Illustrate your procedure to solve the equation.

   $$\frac{\partial T}{\partial t}+U\left(\frac{\partial T}{\partial x}\right)=\left(\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)\right)$$

   On the interval $0<x<1$ with $U=0.2$ and $k=0.5$.

   Boundary conditions are $\frac{\partial T}{\partial x}=0$ at $x=0$ and $x=1$.

   Initial conditions are $T=1$ for $0.3<x<0.7$ and $T=0$ for otherwise.

   Show the matrix used for solution. 8+7=15

6. What is staggered grid ? How is pressure velocity coupling treated using staggering of grid ? Give elaboration about the assumptions. How are momentum equations treated for such problem ? Briefly account for SIMPLE algorithm.

   2+4+2+2=10

[*Turn over*]

7. Give an account of quadratic upwinf differencing scheme with subsequent modifications and assumptions for higher order differencing scheme for convection diffusion problem. 10