Ex/M.Sc/M/B-1.32/36/2017

MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.4 (B-1.32)

(Probability and Stochastic Processes - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations have their usual meanings.)

Attempt Question No. 6 and any *three* from the rest.

1. (a) Define a field and a sigma field.

   (b) Give an example of a field which is not a sigma-field.

   (c) Let Ω be a non-empty space and let $\mathcal{A}$ be a class of subsets of Ω. Define $f(\mathcal{A})$ to be the intersection of all fields containing $\mathcal{A}$.

      (i) Prove that $f(\mathcal{A})$ is a field (it is called the field generated by $\mathcal{A}$) and that $\mathcal{A} \supset f(\mathcal{A})$ and that

[*Turn over*]

$f(A)$ is minimal in the sense that if y is a field and $A \supset g$, then $f(A) \subset g$.

(ii) Show that $f(A)$ is the class of sets of the form :

$\bigcup_{i=1}^{m} \bigcap_{j=1}^{n_i} A_{ij}$, where for each i and j, either $A_{ij}\ \varepsilon\ A$ or the m sets $\bigcap_{j=1}^{n_i} A_{ij}$, $1 \le i \le m$ are disjoint. 4+5+7=16

2. State and prove the $\pi - \lambda$ (Pi-Lambda) theorem. 16

3. State and prove Borel's Normal Number theorem. 16

4. (a) Define a Tail σ-field and a Tail-event. State and prove Kolmogorov's zero-one law on Tail Events.

(b) State and Prove the First Borel-Cantelli Lemma. 8+8

5. Define characteristic function of a random variable. Prove that a characteristic function uniquely specifies the probability distribution of the mother random variable. 16

6. Write down the characteristic function of a standard normal distribution. 2
