MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.4 (B1.18)

(Generalized function and wavelet theory - I)

Full Marks : 50 | Time : Two Hours

The figures in the margin indicate full marks.

Notations have their usual meanings.

Answer any *five* questions.

1. (a) Define a good function and a fairly good function.

(b) Is $\gamma(x)=\begin{cases} 0, & x \le 0 \\ e^{-x^2}, & x > 0 \end{cases}$ a good function ? Justify your answer.

(c) Prove that if $\gamma(x)$ is a good function and $\psi(x)$ is a fairly good function then $\gamma(x)\psi(x)$ is a good function.

4+2+4

[*Turn over*]

2. (a) Define a regular sequence of good functions.

(b) Show that the sequence $\left\{\frac{e^{-\frac{x^2}{n}}}{n}\right\}$ is regular and defines a generalized function $\theta(x)$ connected with the function zero. 2+8

3. (a) Prove that if $\psi(x)$ is a fairly good function and the sequence $\{\gamma_n(x)\}$ is a regular sequence, then $\{\psi(x)\gamma_n(x)\}$ is regular. If $g(x)$ is a generalized function defined by $\{\gamma_n(x)\}$ then for all good functions $\gamma(x)$,

$$\underset{n\to\infty}{Lt}\int_{-\infty}^{\infty}\psi(x)\gamma_n(x)\gamma(x)dx = \int_{-\infty}^{\infty} g(x)\psi(x)\gamma(x)dx.$$

(b) Prove that if $f(x)$ is an ordinary function whose derivative $\frac{df}{dx}$ exists everywhere on $\mathbb{R}$ and

$f(x), \frac{df}{dx} \in \mathcal{K}'(\mathbb{R})$ then

[*Turn over*]

$$f'(x) = \frac{df}{dx},$$

where $f'(x)$ is a generalized function which is the derivative of generalized function $f(x)$. 4+6

4. (a) Prove that if $f(x) \in L'(\mathbb{R})$ and $\int_{-\infty}^{\infty} f(t)\,dt = 1$ then $\underset{n\to\infty}{Lt}\, mf(mx) = \delta(x)$.

(b) Show that

(i) $H'(x) = \delta(x)$

(ii) $|x|' = Sgnx$ 4+(3+3)

5. Prove that

(i) $\underset{t\to 0}{Lt}\, t|x|^{t+1} = 2\delta(x)$

(ii) $\underset{t\to 0}{Lt}\, t|x|^{t-1} = 0$

[*Turn over*]

(iii) $\underset{t \to 0}{Lt} \int_{-\infty}^{\infty} t|x|^{t-1} e^{-i\alpha x} dx = 2$. 5+2+3

6. (a) Prove that if a generalized function is even then its Fourier transform is also even.

(b) If $g(x)$ is a generalized function then $xg(x) = 0$ if and only if $g(x) = c\delta(x)$, c being a constant. 3+7

7. (a) Find the Fourier transform of $\delta(x)$.

(b) Prove that if $\gamma(x)$ is a good function then its Fourier transform $\hat{\gamma}(\alpha)$ is also a good function. 5+5
