Ex/M.Sc/M/B1.16/36/2017

MASTER OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.4 (B1.16)

(Elastodynamics - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

All Symbols have their usual meanings.

Answer Q. No. 1 and any *three* from the rest.

1. Describe the nature of Love surface waves. 2

2. (a) Prove that, in an elastic medium, the displacement vector $\vec{U}$ satisfies the vector equation of motion

$$\rho\frac{\partial^2\vec{u}}{\partial t^2}=(\lambda+\mu)\vec{\nabla}\left(\vec{\nabla}\,.\,\vec{U}\right)+\mu\nabla^2\vec{U}+\rho\vec{F}$$

where $\vec{F}$ is the body force. 8

[*Turn over*]

(b) Apply Helmholtz's theorem to decouple the displacement equation, in absence of body forces, of an isotropic elastic body in the form :

$$\nabla^2\phi = \frac{1}{C_1^2}\frac{\partial^2\phi}{\partial t^2}, \quad \nabla^2\vec{\psi} = \frac{1}{C_2^2}\frac{\partial^2\vec{\psi}}{\partial t^2}.$$ 8

3. Show that $\frac{1}{C^2}\frac{\partial^4 y}{\partial x^2 \partial t^2}$ is the correction term in the equation of flexural vibration of a thin rod when rotatary inertia effect is taken into account. 16

4. Show that the displacement at any point in an infinite elastic medium due to uniform harmonic pressure acting radially on a spherical cavity is given by

$$\frac{p_0 a^3 e^{-i\alpha a}}{\rho w^2 a^2 + 4\mu(i\alpha a - 1)} \cdot \frac{1}{r}\left(i\alpha - \frac{1}{r}\right) e^{i\alpha(r - c_1 t)}$$

where a is the radius of the spherical cavity and p_0 is constant. 16

[*Turn over*]

5. Show that the plane wave solution of the form $\vec{u} = \vec{d}F\left(\vec{r}.\vec{N} - ct\right)$ to the elastodynamic equation is possible if (i) either $\vec{d} = \pm\vec{N}$ or (ii) $\vec{d}.\vec{N} = 0$ and $\mu = \rho c^2$, and explain the cases. 16

6. Deduce the equation of face velocity 'C' with wave length $\frac{2\pi}{\xi}$ for the Love surface wave. 16