Master of Science Examination, 2017

(2nd Year, 1st Semester)

MATHEMATICS

Unit - 3.1

(Advanced Topology)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols have their usual meanings)

Answer Question No. 1 and any *three* from the rest.

1. Give an example of a topological space which is not metrizable. 2

2. (a) Let X be a topological space and $(s_n : n \in D)$ be a net in X. Prove that a point $x_0 \in X$ is a cluster point of $(s_n : n \in D)$ iff some subnet of $(s_n : n \in D)$ converges to x_0. 6

   (b) Define an ultrafilter. Prove that a filter $\hat{\mathcal{F}}$ in X is an ultrafilter in X iff any subset A of X which intersects every member of $\hat{\mathcal{F}}$ belongs to $\hat{\mathcal{F}}$. 1+5

[*Turn over*]

(c) Let X and Y be two topological spaces. Prove that a mapping $f : X \to Y$ is continuous at a point $x_0 \in X$ iff for every net $(s_n : n \in D)$ in X converging to x_0, the net $(f(s_n) : n \in D)$ converges to $f(x_0)$. 4

3. (a) Let (X, τ) be a non-compact topological space and let $X^* = X \cup \{\infty\}$ where ∞ is an element not in X. Denote by τ^*, the family consisting of ϕ, X^*, all members of τ and all subsets U of X^* such that $X^* \setminus U$ is a closed compact subset of X. Prove that (X^*, τ^*) is a compactification of (X, τ). 8

(b) Let X be a Tychonoff space which is not compact. Let $\mathbb{C}$ denote the collection of all T_2-compactifications of X. Then prove that Alexandroff's one point compactification X^* is the minimum element and Stone-Czech compactification $\beta(X)$ is the maximum element in $(\mathbb{C}, \geq)$. 8

4. (a) Define a paracompact space. If every open cover of a topological space X has a closed locally finite refinement then show that X is paracompact. 1+7

[*Turn over*]

(b) Prove that every regular Lindeloff space is paracompact. 4

(c) Prove that a countable compact metric space is totally bounded. Is the converse true ? Justify. 3+1

5. (a) Prove that a complete totally bounded metric space (X, d) is compact. Give two examples to show that both completeness and total boundedness are essential. 4+3

(b) State and prove Stone-Weirstrass Theorem. 8

(c) When is a topological space called metrizable ? 1

6. (a) State and prove Uryshon's Metrization Theorem. 8

(b) Prove that a topological space is uniformizable iff it is completely regular. 8

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