Ex/M.Sc/M/1.5/32/2017

Master of Science Examination, 2017

(1st Year, 1st Semester)

MATHEMATICS

Unit - 1.5

(Differential Geometry)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Notations and Symbols have their usual meanings.

Answer any *five* questions.

1. (a) Find the differential equations of the geodesic for the surface whose first fundamental form is $ds^2 = (du)^2 + (\sin u)^2 (dv)^2$.

   (b) Prove that if A^i and B^i are components of two parallel vector fields, then they are inclined at a constant angle. 6+4

2. (a) Prove that the parametric curves on a surface given by $x^1 = c \sin u \cos v$, $x^2 = c \sin u \sin v$, $x^3 = c \cos u$ are orthogonal. Hence find the Gaussian curvature of this surface.

[*Turn over*]

(b) Find the number of independent components of g_{ij}.

(3+4)+3

3. (a) Define developable surface. Check whether the surface defined by $x^1 = f_1(u^1)$, $x^2 = f_2(u^1)$, $x^3 = u^2$ is developable or not.

(b) If θ is the angle between the parametric curves lying on a surface immersed in E^3, then show that :

$\sin\theta = \frac{\sqrt{a}}{\sqrt{a_{11}\ a_{22}}}$, where $a = |a_{\alpha\beta}|$ (1+4)+5

4. (a) Establish Frenet formulae on a surface.

(b) Prove that $\left\{ \begin{matrix} i \\ i\ j \end{matrix} \right\} = \frac{\partial}{\partial x^j}\left(\log\sqrt{g}\right)$, $g = |g_{ij}| \neq 0$

5+5

5. (a) Define helix. Establish the relation between curvature and torsion of a helix.

(b) Show that the intrinsic derivatives of the fundamental tensors and the kronecker delta are zero. (1+5)+4

6. (a) Prove that for Bertrand mates, the curvature and torsion are related by the relation $ak + b\tau = 1$.

(b) Check whether in V^4, with line element

$$ds^2 = -\left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2 + c^2\left(dx^4\right)^2$$ the vector

$\left(-1, 0, 0, \frac{1}{c}\right)$ is a null vector or not. 7+3

7. (a) Obtain the expression of second fundamental form for a surface and calculate it for the surface given by $r = \left(u\cos v,\ u\sin v,\ c\,v\right)$.

(b) Prove that $R^m_{ijk} + R^m_{jki} + R^m_{kij} = 0$. What is the name of this identity ? 7+3