Master of Science Examination, 2017

(1st Year, 1st Semester)

MATHEMATICS

Unit - 1.4

(General Mechanics)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer any *five* questions.

1. State the principle of least action. What do you mean by Legendre's dual transformation ? Derive Hamilton-Jacobi partial differential equation. 2+3+5=10

2. (a) Starting from D'Alembert's principle, derive Hamilton's principle by using variational method.

   (b) A particle oscillates in a straight line about a centre of force which varies as the distance. Show that the Hamilton's principal function is

$$S=\frac{\sqrt{\mu}}{2}\,\frac{\left[\left(x_0^2+x^2\right)\cos\left\{\sqrt{\mu}\left(t-t_0\right)\right\}-2x\ x_0\right]}{\sin\left\{\sqrt{\mu}\left(t-t_0\right)\right\}}$$

[*Turn over*]

where $x = x_0$ at $t = t_0$ and μx is the force at a distance x. 5+5=10

3. (a) Define generalized momentum corresponding to a generalized co-ordinate. What do you mean by a cyclic co-ordinate ? Show that a cyclic co-ordinate is also absent in the Hamiltonian.

(b) The K.E. and P.E. of a particle are given by

$T = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right)$ and

$V = \frac{A}{x^2} + \frac{A^1}{y^2} + \frac{B}{r} + \frac{B^1}{r^1} + C\left(x^2 + y^2\right)$ where A, A^1, B, B^1 and C are constants and r, r^1 are the distances of the particle from the points whose co-ordinates are $(C,0)$ and $(-C, 0)$ respectively, C being a constant. Show that the system can be converted into Liouville's type.

(1+2+2)+5=10

4. Write down Hamilton's equation of motion in symmetrical form. State the interrelation between poisson bracket and Lagrange's bracket. Prove the Jacobi identity for Poisson bracket. 2+2+6=10

5. (a) Define canonical transformation. Show that the Jacobian of a canonical transformation is unity.

(b) Derive the infinitesimal change of the phase-space variables under infinitesimal canonical transformation.

(c) Examine whether the transformation $Q = \sqrt{q} \cos p$, $P = \sqrt{q} \sin p$ represents a canonical transformation or not. (2+2)+4+2=10

6. (a) Derive velocity and acceleration of a moving particle in 3D using spherical polar co-ordinates.

(b) Establish Euller's dynamical equation of motion.

(3+3)+4=10

7. (a) Show that the time periods of small oscillation of a constraint physical system always lie between the corresponding time periods of the unconstrained system.

(b) Two uniform rods of same mass and of same length $2a$ are freely jointed at a common extremity and rests on two smooth pegs which are in the same horizontal plane. Each rod is inclined at same angle α to the vertical. Show that the time of small oscillation when the join moves in a vertical straight line through the centre of the line joining the pegs is $2\pi\sqrt{\frac{a}{9g}\left(\frac{1+3\cos^2\alpha}{\cos\alpha}\right)}$.

4+6=10