Master of Science Examination, 2017

(1st Year, 1st Semester)

MATHEMATICS

Unit - 1.2

(Real Analysis)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols and Notations have their usual meanings.

Answer Question No. 1 and any *three* from the rest.

1. Give an example of a function for which improper integral exists finitely but the corresponding Lebesgue integral does not exist finitely. 2

2. (a) Define Lebesgue outer measure μ^* and prove that for an interval I, $\mu^*(I)$ is equal to length of I, $\ell(I)$. 1+7

(b) Prove that the union and difference of two Labesgue measurable sets are also Lebesgue measurable. 5

(c) Prove that the Lebesgue measure is continuous from below. 3

[*Turn over*]

3. (a) If E is a countable collection of sets then prove that $R(E)$, the ring generated by E is also countable. Also show that every member of $R(E)$ can be covered by a finite number of sets from E. 6+2

(b) Define a measurable cover. Prove that every set E with σ-finite outer measure has a measurable cover F such that $\mu^*(E) = \bar{\mu}(F)$. 6

(c) Give suitable example to show that a ring may not be a monotone class and a monotone class may not be a ring. 2

4. (a) If $f_n \to f$ in ⓜ then $f_n \to g$ in ⓜ then show that $f = g$ a.e. 4

(b) Give an example to show that convergence in measure in general may not imply convergence almost everywhere. 3

(c) State and prove Egoroff's Theorem. 1+8

5. (a) Prove that for a bounded function f defined on a measurable set E of finite Lebesgue measure, f is Lebesgue integrable if and only if f is measurable. 6

(b) State and prove Dominated convergence Theorem. 1+6

(c) Define a function of bounded variation. Give example to show that a continuous function may not be a function of bounded variation. 1+2

6. (a) If v is a Vitali cover of a bounded set A of real numbers then prove that there is a finite or denumerable number of intervals $\{I_{In}\}$ from v such that $I_i \cap I_j = \phi$ when $i \neq j$ and $\mu^*\left(A - \bigcup_n I_n\right) = 0$. 10

(b) If f is Lebesgue integrable on a Lebesgue measurable set E then prove that for given $\in > 0$, there is a $\delta > 0$ such that for any measurable set $A \subset E$, $\mu(A) < \delta$ implies $\int_A f \, d\mu < \in$. 4

(c) If f is Lebesgue integrable on $[a, b]$ then prove that F defined by $F(x) = \int_a^x f \, d\mu \;\; \forall \, x \in \; [a, b]$ is an uniformly continuous function of bounded variation. 2
