BACHELOR OF SCIENCE EXAMINATION, 2017

(3rd Year, 1st Semester)

PHYSICS (Honours)

Paper - HO-09

Time : **Two hours**

Full Marks : **50**

(25 marks for each group)

Use separate Answer Scripts for each group

GROUP - A

Answer any **five** *questions*

1. Show that

(a)

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$

(b)

$$\delta(a^2 - x^2) = \frac{1}{2|a|}[\delta(x+a) + \delta(x-a)]$$

2+3

2. (a) If $f(x) = u + iv$ is an analytic function and $\vec{A} = \hat{i}v + \hat{j}v$ is a vector, then show that Cauchy-Riemann equations are equivalent to vanishing divergence and curl of $\vec{A}$.

(b) Evaluate $\oint_C \frac{\sin z}{z} dz$ where C is a unit circle.

3+2

3. Evaluate

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5 + 4\cos\theta}$$

5

4. Using Fourier transform, solve the wave equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

subject to the conditions $y(x,0) = F(x)$, $\frac{\partial y}{\partial t}|_{t=0} = 0$

5. Using Laplace transform solve

$$x''(t) + 4x'(t) + 4x(t) = 4e^{-2t}$$

so that $x(0) = -1$, $x'(0) = 4$

6. (a) Expand the function $f(z) = \frac{1}{z(z-1)}$ in terms of Laurent's series.

   (b) Using Cauchy's integral formula evaluate $\oint \frac{z^2}{(z^2-1)} dz$ around a unit circle wi
   centre at $z = 1$

3+

7. Show that if we try to solve Bessel's equation by Forbenius method then the tw
   solutions obtained are not linearly independent.

Ex/FP/I/IX/37/2017

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Group B

Statistical Mechanics - I

Answer any five questions (all carry equal marks) :

1. A gaussian probability distribution is given by,

$$f(x) = A\, e^{(x-\mu)^2/(2\sigma^2)}\,, \quad x \in (-\infty, +\infty).$$

   (a) Find the normalization constant A.

   (b) Find the expectation values $\langle x \rangle$ and $\langle (x - \langle x \rangle)^2 \rangle$. (2+(1+2))

2. A one dimensional discrete random walk has a probability p of moving to the right and probability $q = 1-p$ of moving to the left in any particular step. Let x denote the position of the particle after N steps. Find (i) maximum value x can take, (ii) mean position $\langle x \rangle$, (iii) the root mean square deviation about the mean position $\sqrt{\langle (x - \langle x \rangle)^2 \rangle}$ at the end of N steps. Physically explain what can be said about the position of the particle after N steps. (1+1+2+1)

3. The Hamiltonian for ideal gas is given by,

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\vec{p_i}^2}{2m} \right).$$

   Find the phase space volume between the energy hyper-surfaces $\hat{H} = (E - \Delta/2)$ and $\hat{H} = (E + \Delta/2)$. Hence find the entropy. (2+3)

4. Hamiltonian of a system of N spins, each of which can be either up $S_j = +1$ or down $S_j = -1$, is given by

$$\hat{H} = -\mu_0\, H \sum_{j=1}^{N} S_j$$

Calculate (i) the canonical partition function and (ii) the magnetisation and (iii) magnetic susceptibility of the system. (2+1+2)

5. Consider a one dimensional chain of N Ising spins which can take values $S_i = \pm 1, i = 1, \ldots N$. The Hamiltonian is given by,

$$\hat{H} = -J \sum_{i=1}^{N} S_i \, S_{i+1} \, ,$$

with periodic boundary condition, $S_{N+1} = S_1$. Find the correlation function $\langle S_i \, S_{i+k} \rangle$. Show that it decays exponentially with the distance between the two sites, and obtain its decay constant (correlation length). (2+(2+1))

6. The expression of energy in Ising system is given by,

$$E_I\{s_i\} = -\epsilon \sum_{<i,j>} s_i \, s_j - h \sum_{i=1}^{N} s_i \, ,$$

where the spin variables can take values $s_i = \pm 1$. The sum over $< i, j >$ involves $(\gamma N/2)$ terms, where γ is the number of nearest neighbours of any given site.

(a) Express the energy in terms of number of up-spins N_+, number of (++) pairs N_{++} and the total number of spins N.

(b) State clearly the Bragg-William's approximation, and show that under this approximation the energy depends only on N and N_+. (3+2)

7. Consider an ensemble of $\mathcal{M}$ identical systems at a given temperature T, volume V and chemical potential μ. Let $n_{i,N}$ be the number of ensemble members with N particles and energy E_i. Find an expression for the probability distribution $W\{n_{i,N}\}$. Hence derive the expression for the grand canonical (or macro-canonical) partition function. (2+3)