Bachelor of Science Examination, 2017
(1st Year, 1st Semester)
PHYSICS (Honours)
Paper - HO-01

Time : Two hours | Full Marks : 50

(25 marks for each group)

Use separate Answer Scripts for each group

GROUP - A

Answer any five questions

1. If $\vec{a_i}.\vec{b_j} = \delta_{ij} \quad (i, j = 1, 2, 3)$, show that $\vec{a_1}.(\vec{a_2} \times \vec{a_3}) = [\vec{b_1}.(\vec{b_2} \times \vec{b_3})]^{-1}$. You may use the identity $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = \{\vec{A}.(\vec{B} \times \vec{D})\}\vec{C} - \{\vec{A}.(\vec{B} \times \vec{C})\}\vec{D}$

5

2. (a) A vector function $\vec{f}(x, y, z)$ is not irrotational but the product of $\vec{f}$ and a non zero scalar function $g(x, y, z)$ is irrotational. Show that $\vec{f}$ is $\perp \vec{\nabla} \times \vec{f}$.

   (b) Show that $\delta z = (x^2 + y^2)\mathrm{d}x + 2xy\mathrm{d}y$ is a perfect differential and hence find z.

3+2

3. (a) Find the Fourier Series represenation of

$$f(x) = \begin{cases} 0 & -\pi < x \leq 0 \\ x & 0 \leq x < \pi \end{cases}$$

   (b) Hence show $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

3+2

4. (a) Show that $(\mathrm{AB})^{\mathrm{T}} = \mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}$ where A and B are square matrices.

(b) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew symmetric matrix.

(c) If A and B are two symmetric matrices, then show that AB is symmetric only if A and B commute.

2+2+1

5. Verify Stoke's theorem for the vector field

$$\vec{A} = (3x - 2y)\hat{i} + x^2 z\hat{j} + y^2(z+1)\hat{k}$$

for a plane rectangular area with the vertices at $(0,0)$; $(1,0)$; $(1,2)$ & $(0,2)$ in the $x - y$ plane.

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6. Solve

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} + y = \ln x$$

[Hint : Put $x = e^t$ to get $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = t$]

5

7. If $u = \sin^{-1}\frac{x^2+y^2}{x+y}$ prove using Euler's theoem on homogeneous function

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

5

BACHELOR OF SCIENCE EXAMINATION 2017

(1st Year, 1st Semester)

PHYSICS (HONOURS)

Paper-HO1
GROUP-B

Answer any **five** *questions*

1. A particle of mass m moves in the $x-y$ plane and its position vector is given by $\vec{r} = \hat{i}a\cos(\omega t) + \hat{j}b\sin(\omega t)$, where a, b and ω are constants. Find the force and torque about the origin, acting on the particle. Find also the angular momentum about the origin. Comment on the nature of the force acting on the particle and hence describe the motion of the particle.

(2+1)+1+1

2. (a) Establish that in plane polar coordinates, the radial and transverse components of acceleration are given by

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

(b) A particle moves with $\dot{\theta} = \omega =$ constant and $r = r_o e^{\beta t}$, where r_o and β are constants. Show that for certain values of β, the radial acceleration vanishes while radial velocity increases with time. Justify your answer.

3+2

3. (a) Write down the resistive force in a medium as a function of velocity of a moving object through the medium.

(b) A body of mass m is falling from rest under uniform gravity in a medium whose resistive force is proportional to velocity. Find an expression for the velocity of the body as a function of time. Show that for a short time the effect of the resistive force of the medium can be neglected.

1+(3+1)

4. (a) Using Newton's 2nd and 3rd laws show that the linear momentum of a system of particles is a constant.

(b) Define centre of mass of a system of particles. Find the centre of mass of a thin rectangular plate with sides of length a and b, whose mass per unit area varies as $\sigma = \sigma_o \frac{xy}{ab}$. σ_o is a constant.

2+(1+2)

5. What is meant by a central force? Show that the equation for the path of a particle of mass m under the action of central force is

$$\frac{d^2u}{d\theta^2} + u = \mp\frac{m}{L^2u^2}f\left(\frac{1}{u}\right),$$

where L is a constant of motion and $u = \frac{1}{r}$.

1+4

6. (a) Write down the relation between the angular momentum and angular velocity of a rotating rigid body and hence define the moment of inertia tensor.

(b) Find the inertia tensor for a homogeneous rectangular block of sides a, b and c. M and ρ are the mass and density of the block respectively. Take the origin at one of the corners so that the three adjacent edges lie along the coordinate axes.

2+3

7. A reference frame $S^/$ rotates with respect to another reference frame S with an angular velocity $\vec{\omega}$. If the position, velocity and acceleration of a particle in frame $S^/$ are represented by $\vec{r}, \vec{v}$ and $\vec{a}$, find an expression relating acceleration of the particle in reference frames $S^/$ and S. Explain the terms involved in the expression.

4+1