Ex/FM/5.6/44/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(3rd Year, 1st Semester)

MATHEMATICS (Honours)

Unit - 5.6 (c)

(Mathematical Modelling - I)

Full Marks : 50 Time : Two Hours

All questions carry equal marks.

Use a separate Answer-Script for each part.

Part - I

(30 marks)

Answer any *three* questions. 3×10

1. (a) Define the following :

(i) Critical point,

(ii) Spiral point.

(b) Consider a linear system $\dot{X} = AX$, $X \in \mathbb{R}^2$ with $\delta = |A|$ and $\tau = \text{trace}(A)$. Discuss with suitable figures

[*Turn over*]

the classification of critical points for different values of δ and τ. 4+6

2. (a) Determine the nature of the critical point(s) of the following system and discuss its stability :

(i) $\frac{dx}{dt} = 2x + 4y$; $\frac{dy}{dt} = -2x + 6y$

(ii) $\frac{dx}{dt} = x - y$; $\frac{dy}{dt} = x^2 - 1$.

(b) Define limit cycle of the system

$$\frac{dx}{dt} = P(x,\ y)\ ;\quad \frac{dy}{dt} = Q(x,\ y).$$

(c) Write down the hypothesis for regulation of population growth in density dependent population. 4+3+3

3. What do you mean by functional responses and numerical responses ? Discuss in brief the Holling types functional responses of a prey-predator system. 10

4. Derive the explicit solution of a single species logistic growth model of the form $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$. Also explain the result graphically for different values of K. 10

[*Turn over*]

5. With suitable assumptions, write down the classical Lotka-Volterra prey-predator model and discuss the dynamical behaviour of the system about the equilibrium point. 10

Part - II

(20 marks)

Answer any *two* questions.

6. (a) Write down the three dimensional deterministic host pathogen model with basic assumptions consisting of a host population. 3

(b) Derive the endemic equilibrium point of your formulating mathematical model. 5

(c) Give its stochastic approach and describe the transition state. 2

7. (a) From the model $\frac{dN}{dt} = rN(t)$, where $N(t)$ is the population size at a time t, discuss how population related with the parameter r. 7

(b) Discuss about the regulatory factors of a particular individual in a density dependent population model. 3

[*Turn over*]

8. (a) With suitable assumption formulate a general epidemiological (SIR) model. 4

(b) Define Basic Reproduction Ratio and give its physical classification with your formulated mathematical model. 6

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