BACHELOR OF SCIENCE EXAMINATION, 2017

(3rd Year, 1st Semester)

MATHEMATICS (Honours)

Unit - 5.6 (b)

(Discrete Mathematics - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Use a separate answer-script for each part.

(Symbols have usual meanings, if not mentioned otherwise)

Part - I

(30 marks)

Attempt the questions as follows.

1. Answer any *two* : 5×2=10

(a) Prove that for any graph G with six points, G or $\overline{G}$ contains a triangle.

(b) Define eulerian graph. If the set of lines of a connected graph G can be partitioned into cycles, then prove that G is eulerian.

[*Turn over*]

(c) Define n-cube. Find the number of points and lines in an n-cube.

2. Answer any *two* : 5×2=10

(a) Define the terms coboundary and cocycle. Prove that a cocycle is just a minimal nonzero coboundary.

(b) Prove that there always exists a graph G such that $\kappa(G) = a$, $\lambda(G) = b$ and $\delta(G) = c$ where a, b, c are integers with $0 < a \leq b \leq c$.

(c) Define outerplanar graph. Prove that a maximal outerplane graph with $p \geq 3$ vertices lying on the exterior face contains $p-2$ interior faces.

3. Answer any *one* : 10

(a) If G is a $(p,\ q)$-graph with $p \geq 3$ and for every pair of u and v of nonadjacent points $\deg u + \deg v \geq p$, then prove that G is Hamiltonian.

(b) Prove that any 3-connected planar graph is uniquely embeddable on the plane. 6+4=10

Part - II

(20 marks)

Attempt any *four* questions.

4. Prove that if F_n is the nth Fibonacci number, then

$$F_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right\}$$

for all integer $n \geq 0$. 5

5. Enumerate r-combinations with unlimited repetitions. (assuming there are n distinct objects). 5

6. Prove that

(i) $\left({}^{n}C_0\right)^2 + \left({}^{n}C_1\right)^2 + \ldots + \left({}^{n}C_r\right)^2 + \ldots + \left({}^{n}C_n\right)^2 = {}^{2n}C_n$

(ii) ${}^{m}C_0\,{}^{n}C_0 + {}^{m}C_1\,{}^{n}C_1 + \ldots + {}^{m}C_n\,{}^{n}C_n = {}^{m+n}C_n$ for integers $m \geq n \geq 0$. 5

7. Define devangements. Find a expression for devangements for the set $\{1, 2, \ldots, n\}$. 5

[*Turn over*]

8. Prove that

(i) $F_0 + F_2 + F_4 + ... + F_{2n} = F_{2n+1}$ and

(ii) $F_0^2 + F_1^2 + F_2^2 + ... + F_n^2 = F_n F_n$

where F_n is the nth Fibonacci number. 5

9. Solve the recurrence relation

$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 3$.

Find a particular solution if $a_0 = 0$, $a_1 = 1$ and $a_2 = 10$. 5