BACHELOR OF SCIENCE EXAMINATION, 2017

(Final Year, 1st Semester)

MATHEMATICS (Honours)

Unit - 5.5 (a)

[Mechanics III (Rigid Dynamics)]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings.)

Answer any *five* questions.

1. Show that the moment of inertia of a regular hexagon about any straight line through its centre in the plane of the hexagon is $\frac{5M\,c^2}{24}$, where c is the length of each side, M is the mass of the hexagon. 10

2. (a) If the vertical angle of a cone is 90°, the point at which a generator is a principal axis divides the generator in the ratio 3 : 7. 6

[*Turn over*]

(b) A uniform square lamina is bounded by the axes x and y and the lines $x = 2c,\ y = 2c$ and a corner is cut off by the line $\frac{x}{a} + \frac{y}{b} = 2$. Show that the principal axes at the centre of the square are inclined to the axis of x at angles given by

$$\tan 2\theta = \frac{ab - 2(a+b)c + 3c^2}{(a-b)(a+b-2c)}. \qquad 4$$

3. (a) Show that the centre of suspension and centre of oscillation are convertible. 4

(b) A sphere of radius "a" is suspended by a fine wire from a fixed point at a distance "l" from the centre, show that the time of a small oscillation is given by

$2\pi\sqrt{\frac{5l^2 + 2a^2}{5lg}}\left(1 + \frac{1}{4}\sin^2\frac{\alpha}{2}\right)$, where α, represents the amplitude of vibration. 6

4. (a) A body of mass M is moving in two dimensions. Find the kinetic energy in terms of the motion of the centre of inertia and motion relative to centre of inertia. 4

[*Turn over*]

(b) A circular cylinder whose centre of inertia is at a distance 'c' from the axis rolls on a horizontal plane. If it is just started from a position of unstable equilibrium, show that the normal reaction of the plane when the centre of mass is in its lowest position is $\left[1+\frac{4c^2}{(a-c)^2+k^2}\right]$ times its weight, where 'k' is the radius of gyration about the axis through the centre of mass. 6

5. A uniform rod of length '$2a$' and mass M slides down in a vertical plane its ends begin in contact with two smooth planes, one horizontal and other vertical. It starts from rest at an angle 'α' with the horizon, find the horizontal velocity when the upper end leaves the wall, also find the angular velocity when the rod reaches the horizontal plane. 10

6. A thin uniform rod has one end attached to a smooth hinge and is allowed to fall from a horizontal position. Show that the horizontal strain on the hinge is greatest when the rod is inclined at an angle 45° to the vertical and that the vertical strain is then 11/8 times the weight of the rod. 10

7. (a) A uniform circular board, of mass 'M' and radius 'a', is placed on a perfectly smooth horizontal plane and free to rotate about a vertical axis through its centre ; a man

of mass 'm' walks round the edge of the board whose upper surface is rough enough to prevent his sliding; when he has walked completely round the board to his starting point, show that the board has turned through an angle $\frac{M}{M+2m}4\pi$. 4

(b) A uniform rod of length '$2a$', which has one end attached to a fixed point by a light inextensible string of length $\frac{5a}{12}$, is performing small oscillations in a vertical plane about its position of equilibrium. Find its position at any time, and show that the periods of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}$ and $\sqrt{\frac{a}{3g}}$. 6