Ex/FM/5.1/43/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(Final Year, 1st Semester)

MATHEMATICS (Honours)

Paper - 5.1

(Numerical Methods)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Use a separate Answer-Script for each part.

Part - I

(Marks - 30)

Answer Q. No. 1 and any *three* questions from the rest.

1. (a) Define divided difference of order n. Show that it is a symmetric function of its arguments. 3

(b) If x_t and y_t be the true values approximated respectively by x_a and y_a with small absolute errors e_x and e_y then show that

[*Turn over*]

$e_{xy} \leq |x_a . y_a| \left\{ \left| \frac{e_x}{x_a} \right| + \left| \frac{e_y}{y_a} \right| \right\}$, where e_{xy} is the absolute error in product of x_t and y_t. 3

2. Establish Hermite interpolation formula along with error term. State the applicability of this formula. 8

3. Describe secant method for computing a root of an equation $f(x) = 0$. Find the order of convergence of this method. 8

4. Using the Richardson's extrapolation limit, find $y'(0.05)$ to the function $yx + 1 = 0$ with $h = 0.0128,\ 0.0064,\ 0.0032$.

   Compare your result with the exact value. 8

5. (a) Evaluate $\int_{-1}^{3} |x| \, dx$ analytically and numerically by Simpson's $\frac{1}{3}$ and Weddle's method taking six equal sub-intervals. Which numerical method do give better result and why ? 5

   (b) Find a polynomial $f(x)$ of degree 2 or less such that $f(0) = 1$, $f(1) = 3$ and $f(3) = 55$ by using the

[*Turn over*]

Newton divided difference interpolation formula. Hence find $f(2)$. 3

6. Show that the remainder in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0,\ x_1,\ x_2,\ ...,\ x_n$ is of the form

$$(x-x_0)(x-x_1)(x-x_2),\ ...(x-x_n)\frac{f^{n+1}(\xi)}{\underline{|n+1}},\ \text{where}\ \xi$$

lies between the greatest and smallest of the numbers $x,\ x_0,\ x_1,\ x_2,\ ...,\ x_n$. 8

Part - II

(Marks - 20)

Answer any *two* questions.

6. (a) Use Runge Kutta method to find $y(0.1)$ where $y(x)$ satisfies

$$\frac{dy}{dx} = 2x + y,\ y(0) = 1.$$

(b) Obtain the least square approximations to fit a straight line to the following data

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.447	0.632	0.775	0.894	1

5+5

7. Use Gauss-Jacobi method to solve the following system of linear equations :

$$\begin{aligned} 27x + 6y - z &= 85 \\ 6x + 15y + 27z &= 72 \\ x + y + 54z &= 110 \end{aligned}$$

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8. Describe a method to reduce a $n \times n$ matrix to an upper triangular matrix. Hence obtain the number of operational count. 10