Ex/INT/M/IXS/28/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS (Honours)

Paper - 9S

[Introduction to Linear Algebra & Linear Programming]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Part - I

Answer any *two* questions. 10×2=20

1. (a) Define vector space with examples.

(b) Show that the intersection of two subspaces of a vector space V over a field F is a subspace of V, but their union may not be a subspace of V. 5+5

2. (a) If U, W be two subspaces of a vector space V over a field F, prove that, their linear sum $U+W$ forms a subspace of V and is the smallest subspace of V containing the subspaces U and W.

[*Turn over*]

(b) Determine the subspace of $\mathbb{R}^3$ spanned by the vectors $\alpha = (1, 2, 3)$, $\beta = (3, 1, 0)$. Examine if $\gamma = (2, 1, 3)$, $\delta = (-1, 3, 6)$ are in the subspace. 5+5

3. (a) Prove that the set $S = \{(1, 0, 1), (0, 1, 1), (1, 1, 0)\}$ is a basis of $\mathbb{R}^3$.

(b) Let U and W be two subspaces of a finite dimensional vector space V over a field F. Show that

$$\dim(U + W) = \dim U + \dim W - \dim(U \cap W).$$ 5+5

Part - II

Answer any *three* questions. 10×3=30

4. (a) A firm manufacturing two types of medicines A and B can make a profit of Rs. 20 per bottle of A and Rs. 30 per bottle of B. Both A and B need for their production two essential chemicals C and D. Each bottle of A requires 3 litres of C and two litres of D and each bottle of B requires 2 litres of C and 4 litres of D. The total supply of these chemicals are 210 litres of C and 300 litres of D. Type B medicine contains alcohol and its manufacture is restricted to 65 bottles per month. How many bottles each of A and B should the firm

[*Turn over*]

manufacture per month to maximize its profit of the products ? How much is this profit ? Formulate the problem as the L.P.P. and solve it graphically.

(b) Find the dual of the L.P.P. formulated in question 4(a).

8+2

5. Solve the following LPP :

Maximise $Z = x_1 + 2x_2 + 3x_3 - x_4$

subject to
$$\begin{aligned} x_1 + 2x_2 + 3x_3 \quad &= 15, \\ 2x_1 + x_2 + 5x_3 \quad &= 20, \\ x_1 + 2x_2 + x_3 + x_4 &= 10 \end{aligned}$$

and $x_1, x_2, x_3, x_4 \geq 0$. 10

6. Obtain an optimum basic feasible solution to the following transportation problem :

	W_1	W_2	W_3	W_4	a_i
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
b_j	5	8	7	14	

10

[*Turn over*]

7. Find the optimal assignments for the assignment problem with the following cost matrix :

	I	*II*	*III*	*IV*	*V*
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

10