Ex/INT/M/VIIIS/28/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS (Subsidiary)

Paper - 8S

[Methods of Series Solution of ODE & Special Functions]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer any *five* questions.

1. Find the power series solution of the differential equation :

   $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(3x+2)y=0$ in power of x. Write atleast first four nonzero terms in each series. 10

2. Locate and classify the singular points of the following differential equation.

   $$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$$

   Find the series solution of the above equation. Write atleast

[*Turn over*]

first three nonzero terms in each series. Hence write the solution in terms of elementary functions. 10

3. Prove that

$$P_n(x) = \frac{1}{n!2^n} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n,$$

where $P_n(x)$ is the Legendre polynomial of degree n. Hence find the expressions for P_0, P_1, P_2 and P_3. 10

4. (a) Starting from the relation

$$e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n, \text{ prove that}$$

$$\begin{aligned}\cos(x\cos\theta) &= J_0 - 2J_2\cos 2\theta + 2J_4\cos 4\theta - \cdots \\ \sin(x\cos\theta) &= 2J_1\cos\theta - 2J_3\cos 3\theta + \cdots\end{aligned}$$

where $J_n(x)$ is the Bessel function of first kind and of order n. 7

(b) Write first three terms of Bessel's function of first kind and of order zero $J_0(x)$. Hence find a rough estimate of the first positive zero of it. 3

5. State the orthogonality property of Chebyshev polynomials of first kind. Draw first five Tchebyshev polynomials of first

[*Turn over*]

kind. Find the Tchebyshev series expansion of $\sin\left(\cos^{-1} x\right)$. Write first five terms of the series. 10

6. (a) Expand e^x in Hermite series. Write first two terms of the series. 6

(b) Prove the Rodrigue's formula for Hermite polynomials

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right).$$ 4

7. (a) Starting from the Rodrigue's formula for Legendre polynomials, prove that

$$P_n(x) = \frac{1}{2^n} \sum_{r=0}^{N} \frac{(-1)^r (2n-2r)!\, x^{n-2r}}{r!(n-r)!(n-2r)!},$$

where $P_n(x)$ is the Legendra polynomial of degree n. Determine the appropriate value of N in terms of n, for both even and odd values of n. 5

(b) Prove that $J_{-n} = (-1)^n J_n$ for integral values of n,

where $J_n(x)$ is the Bessel function of first kind and of order n. 5

———