BACHELOR OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS (Honours)

Paper - 7S

[Vector Calculus]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer any *five* questions.

1. (a) Prove that the necessary and sufficient condition for a vector function $\overline{f}(t)$ to be constant is $\frac{d}{dt}\overline{f}(t)=0$. 5

   (b) Prove that

   $$\frac{d}{dt}\left(\overline{a}.\overline{b}\right)=\overline{a}.\frac{d\overline{b}}{dt}+\frac{d\overline{a}}{dt}.\overline{b}\ .$$ 5

2. (a) If $\overline{\alpha}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k}$ and $\hat{\beta}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$ where $\hat{i},\hat{j},\hat{k}$ have their usual meanings, find

[*Turn over*]

$\frac{d}{dt}\left(\bar{\alpha}\times\frac{d\bar{\beta}}{dt}\right)$ at $t = 2$. 5

(b) If $\bar{r} = \bar{a}e^{nt} + \bar{b}e^{-nt}$ where $\bar{a}$ and $\bar{b}$ are constant vectors and n is constant, then show that $\frac{d^2\bar{r}}{dt^2} - n^2\bar{r} = 0$. 5

3. (a) Find the directional derivative of $\phi = xy^2z + 4x^2z$ at $(-1, 1, 2)$ in the direction $(2i + j - 2k)$. 5

(b) If $u = x^3 + 3yz^2$, find grad u. 5

4. (a) Let $\vec{u}$ and $\vec{v}$ be two vector point functions, then show that $div(\vec{u}\times\vec{v}) = \vec{v}.curl\ \vec{u} - \vec{u}.curl\ \vec{v}$. 5

(b) Define irrotational vector. Prove that the vector

$$\vec{a} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$$

is irrotational vector. 5

5. (a) If $\vec{r}$ be the position vector of a point $P(x, y, z)$ from the origin, then find $div\ \vec{r}, curl\ \vec{r}, \nabla^2\left(\frac{1}{r}\right)$. 5

[*Turn over*]

(b) Evaluate $\int_C F.dr$ where $F = x^2y^2\, i + y\, j$ the curve C is given by $C : y^2 = 4x$ in the *xy*-plane from (0, 0) to (4, 4). 5

6. State Stokes' theorem. Verify Stokes' theorem for $F(x, y, z) = (2x - y)i - yz^2 j - y^2 zk$ where S is the upper-half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 10

7. State Divergence theorem. Verify the theorem for the function $F = (2x - z)i + x^2 y\, j - z^2 xk$ over the region bounded by the surface $x = 0,\ x = 1;\ y = 0,\ y = 1;\ z = 0,$ $z = 1$.. 10
