Ex/INT/M/3.1/27/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

MATHEMATICS (Honours)

Paper - 3.1

(Mechanics - II)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols / Notations have their usual meanings.)

Answer Q. No. 10 and any *six* questions from the rest.

1. The acceleration of a particle moving in a plane curve is resolved into two components, one component is parallel to the initial line and the other component is along the radius vector. Prove that these components are

$$-\frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) \text{ and } \frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2+\frac{\cot\theta}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)$$ 8

2. A particle moves in a plane under a force which is always perpendicular and towards a fixed straight line. The magnitude of the force is $m\mu \div (\text{distance from the line})^2$,

[*Turn over*]

where m is the mass of the particle and $\mu(>0)$ is a constant. If initially it be at a distance $2a$ from the line and be projected with a velocity $\sqrt{\frac{\mu}{a}}$ parallel to the line, prove that the path of the particle is a cycloid. 8

3. A particle of mass m is moving under the action of central force mF in a medium which exerts a resistance equal to $k \times (velocity)^2$ per unit mass. Write the equation of motion of the particle along radial and cross-radial directions. Hence show that the differential equation of the path of the particle is

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h_0^2 u^2} e^{2ks},$$

where s is the length of the arc described in time t and h_0 is the initial moment of momentum per unit mass about the centre of force. 8

4. A particle is moving under the action of a central force $\frac{\mu}{r^3}$ per unit mass. It is projected from an apse at a distance a from the centre of force with a velocity equal to $\sqrt{2}$ times

[*Turn over*]

the velocity in a circle of radius a. Show that the polar equation of its path is

$$r\cos\frac{\theta}{\sqrt{2}} = a\,.$$ 8

5. A particle is free to move on a smooth vertical wire of radius a. It is projected from the lowest point with a velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time.

$$\sqrt{\frac{a}{g}}\log\left(\sqrt{5}+\sqrt{6}\right).$$ 8

6. A particle of mass m projected with a velocity u, is acted upon by a force which produces a constant acceleration f in the plane of motion inclined at a constant angle α with the direction of motion. Obtain the intrinsic equation of the path of the particle. Show also that the particle will be moving in the opposite direction to that of projection at time

$$\frac{u}{f\cos\alpha}\left(e^{\pi\cot\alpha}-1\right).$$ 8

[*Turn over*]

7. Find the expressions of velocities and accelerations of a particle moving in a plane with respect to a set of mutually perpendicular rotating axes. 8

8. (a) A particle of mass m describes an ellipse under the action of the central force $\frac{m\mu}{r^2}$ with a focus as the centre of force, where $\mu(>0)$ is a constant. Prove that the velocity at the end of the minor axis is the geometric mean of the velocities at the ends of any diameter.

   (b) A heavy particle slides down a rough cycloid whose base is horizontal and vertex downwards. Show that if it starts from rest at the cusp and comes to rest at the vertex, then $\mu^2 e^{\mu\pi} = 1$, where $\mu(>0)$ is the coefficient of friction. 4+4=8

9. A particle whose mass increases through condensation of moisture at a constant rate $\frac{m_0}{\tau}$, where m_0 is the initial mass of the particle and τ is a constant, moves freely under gravity. The particle is projected from the origin of a rectangular coordinate system whose x-axis is horizontal and y-axis is vertically upwards. Prove that the coordinates of the particle when its mass is m are given by

[*Turn over*]

$$x = u\tau \log\left(\frac{m}{m_0}\right),$$

$$y = \frac{1}{4}g\tau^2\left\{1-\left(\frac{m}{m_0}\right)^2\right\}+\left(v\tau+\frac{1}{2}g\tau^2\right)\log\left(\frac{m}{m_0}\right),$$

where u, v are the components of velocities of the ball along x-axis and y-axis respectively at the origin. 8

10. Consider a particle of mass m moving in a plane under the action of any force. Prove that the resultant velocity of the particle is always acting along the tangent to the path of the particle. 2

Or,

Derive the expression of the escape velocity of a particle when it is projected from the surface of the Earth. 2