Ex/INT/M/7Stat/31/2017

BACHELOR OF SCIENCE EXAMINATION, 2017

(2nd Year, 1st Semester)

STATISTICS

Paper - 7Stat

[Inference - I]

Full Marks : 50 | Time : Two Hours

Answer any *five* questions.

Each questions carries 10 marks.

1. Define Bias of an Estimator. When an estimator is called (a) unbiased (b) Asymptotically unbiased.

   Let X~Binomial (n, θ), let n be known and the unknown parameter θ be estimated by the estimator $T(X) = \dfrac{X + \sqrt{n}/2}{n + \sqrt{n}}$, is $T(X)$ unbiased for θ ? Is $T(X)$ asymptotically unbiased for θ ?

2. Define a sufficient statistic. Show that if $X_1, \cdots, X_n$ are i.i.d.

[*Turn over*]

(independent and identically distributed) Poisson (λ), $(\lambda > 0, \text{unknown})$ then $\sum_{1}^{n} X_i$ is sufficient for λ.

3. State and prove Fisher-Neyman factorisation theorem (for discrete distributions only.)

4. Define a Uniformly Minimum Variance Unbiased Estimator (UMVUE). State and prove Rao-Cramer Inequality.

5. Given a random sample of size n from a population with mean 10 and unknown variance σ^2, prove that $\frac{1}{n}\sum_{i=1}^{10}(X_i - 10)^2$ is unbiased for σ^2 ?

6. If $X_1, \cdots, X_n$ is a random sample of size n from an exponential (θ) population, show that $\overline{X}$ is consistent for θ.

7. Using Neyman-Pearson lemma, find a Most Powerful test (use $\alpha = 5\%$) for testing $H_0 : \sigma^2 = 1$ vs $H_1 : \sigma^2 = 2$, with an iid sample of size n from a Normal population with mean 0 and variance σ^2.
