# Bachelor of Science Examination, 2017

**(1st Year, 1st Semester)**

**MATHEMATICS (Honours)**

**Unit - 1.2**

**(Geometry)**

Full Marks : 50 Time : Two Hours

*The figures in the margin indicate full marks.*

Use a separate Answer-Script for each part.

(Notations/Symbols have their usual meanings.)

## Part - I

(20 Marks)

Answer any *two* questions.

1. (a) Show that the necessary and sufficient conditions that the general equation of second degree in $x$, $y$ should represent a pair of straight lines is

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0.$$

[*Turn over*]

(b) Prove that the transformation of rectangular axes which converts $\frac{X^2}{p}+\frac{Y^2}{q}$ into $ax^2+2hxy+by^2$ will convert $\frac{X^2}{p-\lambda}+\frac{Y^2}{q-\lambda}$ to

$$\frac{ax^2+2hxy+by^2-\lambda\left(ab-h^2\right)\left(x^2+y^2\right)}{1-(a+b)\lambda+\left(ab-h^2\right)\lambda^2}.$$ 6+4

2. (a) Show that the equation

$2x^2-3xy-2y^2+7x-9y=0$ represents a central conic and hence reduce it into canonical form.

(b) Define confocal conics. Show that through every point in the plane of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, two confocal conics can be drawn, one is an ellipse and the other is a hyperbola. 5+5

3. (a) If the equation of the conic is $ax^2+2hxy+by^2+c=0$, show that the origin is the centre.

(b) Show that the straight lines joining the origin to the points of intersection of the curves

$$ax^2 + 2hxy + by^2 + 2gx = 0$$

and $a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$ will be at right angle if $g(a' + b') = g'(a + b)$.

(c) If a focal chord of an ellipse makes an angle $\alpha$ with the major axis, show that the angle between the tangents at its extremities is

$$\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right).$$ 2+3+5

**Part - II**

(30 Marks)

Answer any *two* from Q. No. 4 to Q. No. 6 and

and *one* from Q. No. 7 to Q. No. 8.

4. (a) Show that the equation to the plane containing the straight line $\frac{y}{b} + \frac{z}{c} = 1$, $x = 0$ and parallel to the straight line $\frac{x}{a} - \frac{z}{c} = 1$, $y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$ and if $2d$ be

[*Turn over*]

the shortest distance between the lines, then show that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$. 6

(b) Perpendiculars PL, PM, PN are drawn from the point $P(a,\ b,\ c)$ to the co-ordinate planes. Show that the equation of the plane LMN is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=2$. 4

5. (a) Show that the surface generated by a straight line which intersects the straight lines

$y=0,\ z=c;\ x=0,\ z=-c$ and the curve

$z=0,\ xy+c^2=0$ is $z^2-c^2=xy$. 5

(b) A sphere of constant radius $2k$ passes through the origin and cuts the axes at A, B, C. Show that the locus of the centroid of the tetrahedron OABC is the sphere $x^2+y^2+z^2=k^2$. 5

6. (a) A variable line always intersects the line $z=0,\ x=y$ and the circles

$x=0,\ y^2+z^2=d^2;\ y=0,\ z^2+x^2=d^2$.

[*Turn over*]

Show that the equation to its locus is

$$(x+y)^2\left\{z^2+(x-y)^2\right\}=d^2(x-y)^2.$$ 6

(b) Prove that the locus of the point which is equidistant from the lines $y=mx,\ z=c$ and $y=-mx,\ z=-c$ is the surface $mxy+\left(1+m^2\right)cz=0$. 4

7. (a) Find the equation of the cylinder whose generators are parallel to the line $x=-\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1,\ z=3$. 5

(b) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ whose vertex is $P$, by the plane $z=0$ is a rectangular hyperbola. Show that the locus of $P$ is

$$\frac{x^2+y^2}{a^2+b^2}+\frac{z^2}{c^2}=1.$$ 5

8. (a) A sphere of constant radius $r$ passes through the origin cuts the axes in A, B, C. Prove that the locus of the

[*Turn over*]

foot of the perpendicular from origin to the plane ABC is given by $\left(x^2+y^2+z^2\right)^2\left(x^{-2}+y^{-2}+z^{-2}\right)=4r^2$. 5

(b) Show that the condition that the plane $ax+by+cz=0$ may cut the cone $yz+zx+xy=0$ in perpendicular lines is $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 5