BACHELOR OF SCIENCE EXAMINATION, 2017

(1st Year, 1st Semester)

MATHEMATICS (Honours)

Unit - 1.1

(Calculus)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Use a separate Answer-Script for each part.

(Notations and Symbols have their usual meanings.)

Part - I (30 Marks)

Answer any *three* questions. 10×3=30

1. (a) Does $\lim_{x\to 0}\left(\sin\frac{1}{x}+x\sin\frac{1}{x}\right)$ exist ?

(b) Let the function f be defined by

$$f(x)=\begin{cases}\dfrac{1-\cos x}{x^2}, & \text{for } x\neq 0\\ 1, & \text{for } x=0\end{cases}$$

Is f continuous at $x=0$? Explain.

[*Turn over*]

(c) Let $f(x)=\begin{cases} x^m \sin\frac{1}{x}, & \text{when } x\neq 0 \\ 0, & \text{when } x=0 \end{cases}$

Then show that $f(x)$ is derivable at $x=0$. Also determine m when $f'(x)$ is continuous at $x=0$.

2½+2½+5

2. (a) If $f(x)=\tan x$, then show that

$$f^n(0)-{}^nc_2 f^{n-2}(0)+{}^nc_4 f^{n-4}(0)-\ldots=\sin n\pi/2$$

(b) State Rolle's theorem. Use this theorem to show that the polynomial equation

$c_n x^n + c_{n-1}x^{n-1}+\ldots+c_1 x+c_0=0$ has at least one root between 0 and 1, if $c_0+\frac{c_1}{2}+\frac{c_2}{3}+\ldots+\frac{c_n}{n+1}=0$. 4+6

3. (a) State Euler's theorem for homogeneous function of two variables.

If $V=\sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then

[*Turn over*]

prove that

$$x^2\frac{\partial^2 v}{\partial x^2}+2xy\frac{\partial^2 v}{\partial x\partial y}+y^2\frac{\partial^2 v}{\partial y^2}=\frac{\tan v}{12}\left(\frac{13}{12}+\frac{\tan^2 v}{12}\right).$$

(b) Use mean value theorem of appropriate order to prove that

$$x-\frac{x^2}{2}<\log(1+x)<x, \quad \text{for all } x>0.$$ 6+4

4. (a) Examine the continuity of the function

$$f(x,\ y)=\begin{cases}\dfrac{x^4+y^4}{x-y}, & \text{when } x\neq y\\ 0, & \text{when } x=y\end{cases}$$

at (0, 0).

(b) Let $f(x,\ y)=\begin{cases}x\sin\dfrac{1}{y}+y\sin\dfrac{1}{x}, & \text{when } xy\neq 0\\ 0, & \text{when } xy=0\end{cases}$

then show that $\lim\limits_{(x,\ y)\to(0,\ 0)} f(x,\ y)=0$

but the repeated limits do not exist. 5+5

5. (a) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$.

(b) State and prove fundamental theorem of Integral Calculus. 4+6

Part - II (20 Marks)

Attempt any *four* questions. 5×4=20

6. Define asymptote of a curve. Find the asymptotes of the curve $\left(x^2-y^2\right)^2-4y^2+y=0$.

7. For any curve $r=f(\theta)$. Prove that $\frac{ds}{dr}=\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}$.

8. Define the pedal equation of a curve with respect to a fixed point on the plane of the curve. Find the pedal equation of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ w.r.t origin.

9. Find the surface area of the solid generated by revolution of the cardioide $r=a(1+\cos\theta)$ about the initial line.

[*Turn over*]

10. If p_1 and p_2 be the perpendiculars from the origin on the tangent and normal respectively at any point $(x,\ y)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Then show that $4p_1^2 + p_2^2 = a^2$.

11. Find the radius of curvature at any point P of the catenary $y = c\ \cosh\left(\frac{x}{c}\right)$ and show that $PC = PG$, where C is the centre of curvature at P and G is the point of intersection of the normal at P with X-axis.
