BACHELOR OF SCIENCE EXAMINATION, 2017

(1st Year, 1st Semester)

MATHEMATICS (Subsidiary)

Paper - IIS

(Algebra-1)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer any *five* questions.

1. (a) State and prove DeMoivre's theorem. 5

(b) If z is a variable complex number such that an amplitude of $\frac{z-i}{z+1}$ is $\frac{\pi}{4}$. Show that the point z lies on a circle in the complex plane. Also find the radius and centre of the circle. 5

2. (a) Define a^z where 'a' is a nonzero complex number and z is any complex number. If a, b, z are complex numbers such that $ab \neq 0$, prove that $(ab)^z = a^z.b^z$.

[*Turn over*]

Is the principal value of $(ab)^z$ equal to the product of the principal values of a^z and b^z? Justify.

(b) If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$, prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \tfrac{3}{2}.$$

5+5=10

3\. (a) Find the principal value of $\log(1 + \cos 2\theta + i\sin 2\theta)$, $\frac{\pi}{2} < \theta < \pi$. 3

(b) If n be a positive integer and

$(1+z)^n = p_0 + p_1 z + p_2 z^2 + \cdots + p_n z^n$ then show that

$p_0 - p_2 + p_4 - \cdots = 2^{n/2} \cos\dfrac{n\pi}{4}$. 2

(c) If a, b, c, x, y, z be all real numbers and $a^2 + b^2 + c^2 = 1$, $x^2 + y^2 + z^2 = 1$, then show that $-1 \le ax + by + cz \le 1$. 5

4\. (a) Show that the p-series $\sum \dfrac{1}{n^p}$ converges only when $p > 1$.

[*Turn over*]

Also show that the harmonic series $\sum \frac{1}{n}$ diverges. 5

(b) Test the converges of the following series

$$1+\frac{1}{2}.\frac{1}{3}+\frac{1.3}{2.4}.\frac{1}{5}+\frac{1.3.5}{2.4.6}.\frac{1}{7}+\cdots$$ 5

5. (a) Find the region of convergence of the series $\frac{x}{1^2}+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\cdots$. 5

(b) Test the convergence of the series

$$\left(\frac{2^2}{1^2}-\frac{4}{1}\right)+\left(\frac{3^3}{2^3}-\frac{5}{2}\right)^2+\left(\frac{4^4}{3^4}-\frac{6}{3}\right)^3+\cdots.$$ 5

6. (a) Write the Descarte's rule of signs. Apply the rule to examine the nature of roots of the equation $x^7+x^5-x^3=0$. 5

(b) Solve $x^3-3x-1=0$ by Cardan's method. 5

[*Turn over*]

7. (a) If α be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0, \; (d \neq 0)$, show that $\alpha = -\frac{8d}{3c}$. 5

(b) If a, b, c be positive real numbers, prove that

$$(ab + bc + ca)\left(a.b^{-1} + b.c^{-1} + c.a^{-1}\right) \geq (a + b + c)^2.$$

5
