BACHELOR OF SCIENCE EXAMINATION, 2017

(1st Year, 1st Semester)

MATHEMATICS (Subsidiary)

Paper - 1S

(Calculus - I)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings)

Answer any *five* questions.

1. (a) Show that the $\lim_{x\to 0} \frac{1}{1+e^{1/x}}$ does not exist.

   (b) State Leibnitz's theorem on Successive - Differentiation. Using this theorem prove that if $y = \cos\left(m \sin^{-1} x\right)$, then $\left(1-x^2\right) y_{n+2} - \left(2n+1\right) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0$.

   Also find y_n for $x = 0$. 4+6=10

2. (a) Let f be real valued function defined over [–1, 1] such

[*Turn over*]

that $f(x)=\begin{cases} x\cos\dfrac{1}{x}, & x\neq 0 \\ 0 \quad , & x=0 \cdot \end{cases}$

Does the mean value theorem hold for f in [–1, 1] ?

(b) Use mean value theorem of appropriate order to prove that $\sin x > x-\dfrac{x^3}{3!}$, when $0<x<\pi/2$.

(c) Evaluate the $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}}$. 2+4+4=10

3. (a) Test the convergence of the following :

(i) $\displaystyle\int_1^{\infty}\frac{dx}{x(1+x)}$

(ii) $\displaystyle\int_0^{1}\frac{dx}{\sqrt{1-x^2}}$

(b) Show that $\displaystyle\int_0^{\pi/2}\frac{\sin^m x}{x^n}\,dx$ converges for $n<m+1$. 5+5

[*Turn over*]

4. (a) If $I_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$, then prove that $I_n = \frac{1}{n-1} - I_{n-2}$.

(b) Prove that $B(x, y) = \frac{\Gamma(x).\Gamma(y)}{\Gamma(x+y)}$; for $x, y > 0$.

5+5=10

5. (a) Find the asymptotes of the curve

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + y^2 - x^2 + 2x - 3y - 1 = 0.$$

(b) Find the radius of curvature at the origin of

$$y^2 = \frac{x^2(a+x)}{(a-x)}.$$

5+5=10

6. (a) Show that the minimum value of $\frac{(2x-1)(x-8)}{x^2-5x+4}$ is greater than its maximum value.

(b) If
$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0; \end{cases}$$

then show that $f(x)$ is continuous at $x = 0$. 6+4=10

[*Turn over*]

7. (a) Find the values of a and b so that

$$\lim_{x \to 0} \frac{a \sin 2x - b \sin x}{x^3} = 1.$$

(b) State Rolle's theorem. What is its geometrical interpretations ?

(c) In Cauchy Mean value theorem, $\varphi(x) = \sin x$, $\psi(x) = \cos x$. Show that θ is independent of both x and h and equal to $\frac{1}{2}$. 4+2+4=10
