BACHELOR OF SCIENCE EXAMINATION, 2017

(1st Year, 1st Semester)

MATHEMATICS (Subsidiary)

Paper - 3 S

(Analytical Geometry)

Full Marks : 50 Time : Two Hours

Use separate Answer-Script for each Group.

The figures in the margin indicate full marks.

(Notations / Symbols have their usual meanings)

Group - A

(Marks : 20)

Answer any *two* questions. 10×2=20

1. (a) Transform the equation $2x^2 - xy + y^2 + 2x - 3y + 5 = 0$ to new axes of x and y given by the straight line $4x + 3y + 1 = 0$ and $3x - 4y + 2 = 0$ respectively.

   (b) Find the angle through which the axes are to be rotated so that the equation $x\sqrt{3} + y + 6 = 0$ may be reduced to the form $x\, r = c$. Also determine the value of c.

5+5=10

[*Turn over*]

2. (a) Show that the area of the triangle formed by the straight lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is $\sqrt{h^2 - ab} \big/ \left(am^2 - 2hlm + bl^2\right)$.

(b) Find the condition that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ may coincide with one of the straight lines given by $px^2 + 2qxy + qy^2 = 0$. 5+5=10

3. (a) If the pole of the straight line with respect to the circle $x^2 + y^2 = a^2$ lies on $x^2 + y^2 = k^2a^2$, then prove that the straight line will touch the circle $x^2 + y^2 = a^2 \big/ k^2$.

(b) Show that the locus of the poles of tangents to the parabola $ay^2 + 2b^2x = 0$ with respect to the ellipse $x^2 \big/ a^2 + y^2 \big/ b^2 = 1$ is the parabola $ay^2 - 2b^2x = 0$.

5+5=10

Group - B

(Marks : 30)

Answer any *three* questions. 10×3=30

4. (a) Find the angle between the straight lines whose direction ratios are (5, –12, 13) and (–3, 4, 5).

[*Turn over*]

(b) If (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of two perpendicular straight lines, then show that the direction cosines of the straight line perpendicular to both of them are

$\pm(m_1n_2 - m_2n_1), \pm(n_1l_2 - n_2l_1), \pm(l_1m_2 - l_2m_1)$ 3+7

5. (a) Find the equation of the plane passing through three points (2, 2, –1) ; (3, 4, 2) and (7, 0, 6).

(b) Find the equation of the plane which passes through the point (2, 1, –1) and is orthogonal to each of $x - y + z = 1$ and $3x + 4y - 2z = 0$. 5+5

6. (a) Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$.

(b) Find the values of b and c for which the straight line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+3}{3}$ lies on the plane $9x + by + cz = 30$. 5+5

[*Turn over*]

7. (a) Find the condition that the straight lines $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$; $\frac{x}{a\alpha}=\frac{y}{b\beta}=\frac{z}{c\gamma}$ and $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ are coplanar.

(b) Find the shortest distance between the straight lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$

5+5

8. A sphere S has points (0, 1, 0) and (3, –5, 2) as the opposite ends of a diameter. Find the equation of the sphere on which the intersection of the plane $5x-2y+4z+7=0$ with S is a great circle. 10