Ex/Prod/Math/T/211/2017(Old) (S)

BACHELOR OF ENGINEERING IN PRODUCTION ENGINEERING EXAMINATION, 2017

(2nd Year, 1st Semester, Supplementary)

MATHEMATICS - VS (OLD)

Time : Three hours | Full Marks : 100

Answer ***any six*** questions.

Four marks are reserved for neatness.

(Notations have their usual meanings.)

1. a) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$.

   b) If f(z) is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2 .$$

8+8

2. a) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that f(z) = u + iv is analytic.

   b) Determine which of the following function are analytic

   (i) e^z ; (ii) SinZ.

8+8

3. a) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the paths $y = x^2$.

[Turn over

b) Prove that $\oint_C \frac{dz}{z-a} = 2\pi i$

c) Prove that $\oint_C (z-a)^n dz = 0$ [n is an integer] where C is the circle $|z-a| = r$ 5+5+6

4. a) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $|z| = 1/2$.

b) Evaluate $\oint_C \frac{3z^2+z}{z^2-1} dz$, wehre C is the circle $|z-1| = 1$.

8+8

5. a) Find the sum of the residues of the function $f(z) = \frac{SinZ}{ZCosZ}$ at its poles inside the circle $|z| = 2$,

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+Cos\theta}$. 8+8

6. a) Find the power series solution of the differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \text{ in power of x about } x = 0.$$

b) Prove that $e^{\frac{x}{2}(t-1/t)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$ 9+7

7. a) Find the powre series solution of the differential equation

$$x^2\frac{d^2y}{dx^2} + (x^2+x)\frac{dy}{dx} + (x-9)y = 0$$

in power of x about $x = 0$.

b) Prove that $\left[J_{1/2}(x)\right]^2 + \left[J_{-1/2}(x)\right]^2 = \frac{2}{\pi x}$. 10+6

8. a) Prove that $\int_{-1}^{1} P_m(n)P_n(n)dn = 0$, when $m \neq n$

$$= \frac{2}{2n+1} \text{ when } m = h.$$

b) Prove that $(2n+1)P_n(n) = P_{n+1}'(x) - P_{n-1}'(x)$ 10+6

9. a) Prove that $[J_o(n)]^2 + 2[J_1(n)]^2 + 2[J_2(n)]^2 + \cdots = 1$.

b) Prove that $x^4 = \frac{1}{35}\left[8P_4(n) + 20P_2(n) + 7P_0(n)\right]$.

c) Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial. 6+5+5