Bachelor of Engineering in Production Engineering Examination, 2017

(1st Year, 2nd Semester)

Mathematics - III

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Notations/Symbols have their usual meanings.

Answer ***any five*** questions :

1. a) If $\frac{d\bar{u}}{dt} = \bar{w} \times \bar{u}$ and $\frac{d\bar{v}}{dt} = \bar{w} \times \bar{v}$ then prove that

$$\frac{d}{dt}(\bar{u} \times \bar{v}) = \bar{w} \times (\bar{u} \times \bar{v})$$

b) The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move ? 5

2. a) If $u\bar{F} = \bar{\nabla}v$, where u, v are scalar fields and $\bar{F}$ is a vector field, evaluate

$$\bar{F} \cdot \bar{\nabla} \times \bar{F}$$ 5

[Turn over

b) Compute the line integral $\int_C (y^2dx - x^2dy)$ about the triangle whose vertices are (1, 0), (0, 1) and (–1, 0). 5

3. a) Verify Green's theorem for $\int_C (y - \text{Sin}x)dx + \text{Csx } dy$ where C is the boundary of the region bounded by the lines $y = 0, \ x = \pi/2$ and $y = \frac{2}{\pi}x.$ 7

b) If the vector functions $\overline{f}$ and $\overline{g}$ are irrotational, show that $\overline{f} \times \overline{g}$ is solenoidal. 3

4. a) State Divergence theorem.

Evaluate $\iint_S \{x^2dydxz + x^2dzdx + 2z(xy - x - y)dxdy$ where S is the surface of the cube $0 \le x \le 1, \ 0 \le y \le 1$ and $0 \le z \le 1.$ 5

b) If $\overline{r} = x\hat{i} + y\hat{i} + z\hat{k}$ and $r \neq 0$ then show that

i) $\overline{\nabla}\left(\frac{1}{r^2}\right) = -\frac{2\overline{r}}{r^4}$

ii) $\overline{\nabla} \cdot \left(\frac{\overline{r}}{r^2}\right) = \frac{1}{r^2}$ 5

5. Verify Stoke's theorem for $\overline{f} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b.$ 10

6. a) Find the Fourier transform of

$$\begin{aligned} f(x) &= 1 - x^2 \quad \text{if} \quad |x| < 1 \\ &= 0 \quad \text{if} \quad |x| > 1 \end{aligned}$$ 5

b) Find $\overline{F}_S(S)$ and $\overline{F}_C(S)$ are Fourier Sine transform and Cosine transform of $f(x)$ respectively, then show that

$$\overline{F}_S[f(x)\text{Sin}ax] = \frac{1}{2}[\overline{F}_C(S - a) - \overline{F}_C(S + a)]$$

7. a) If $Z(u_n) = \overline{u}(z),$ then show that 5

b) Prove that

$$Z(\text{Sin}\theta) = \frac{Z\text{Sin}\theta}{Z^2 - 2ZC_S\theta + 1} \text{ and}$$

$$Z(\text{Sin}\theta) = \frac{Z\text{Sin}\theta}{Z^2 - 2ZC_S\theta + 1}$$ 5

PART - B

Answer ***any five*** questions :

8. Obtain a fourier series for the function f(x) given by

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \le x \le 0$$

$$1 - \frac{2x}{\pi}, 0 \le x \le \pi$$

Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. 10

9. Obtain a Fourier series for the function

$$\begin{aligned} f(x) &= \pi x \quad , \ 0 \le x \le 1 \\ &= \pi(2-x), \ 1 < x \le 2 \end{aligned}$$

10

10. If $f(x) = x, 0 < x \le \frac{\pi}{2}$

$$= \pi - x, \frac{\pi}{2} < x < \pi$$

show that

i) $f(x) = \frac{4}{\pi}\left[\text{Sinx} - \frac{\text{Sin3x}}{3^2} + \frac{\text{Sin5x}}{5^2} - \cdots\right]$

ii) $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\frac{\text{Cos2x}}{1^2} + \frac{\text{Cos6x}}{3^2} + \frac{\text{Cos10x}}{5^2} + \cdots\right]$ 5+5

11. The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the spring is released from rest.

Derive an expression for the displacement of the spring at subsequent times and show that the midpoint of the string always remains at rest. 10

12. A rod of length l with insulated sides is initially at a uniform temperature u_o. It's ends are suddenly cooled to 0^oC and are kept at that temperature. Find the temperature function u(x,t), where x is the distance from one end at time t. 10

13. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t. 10

14. Let u be a harmonic function in $0 \le x \le a,\ 0 \le y \le b$ in the xy plane, satisfying $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial u^2} = 0$ with u(0, y)=0, u(a, y)=0, u(x, b)=0 and u(x, 0)=f(x). Determine u(x, y). 10