Bachelor of Production Engineering Examination, 2017(S)

(1st Year, 1st Semester Supplementary)

MATHEMATICS II

Time : Three hours Full Marks : 100

(Symbols/ Notations have their usual meanings)

*Answer **any five** questions*

1.(a) Solve the following differential equations:

(i) $ydx - xdy + \log x\, dx = 0$

(ii) $(1+xy)ydx + (1-xy)xdy = 0$

(iii) $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(iv) Show that the differential equation $(ax+hy+g)dx + (hx+by+f)dy = 0$ is the differential equation of a family of conics.

(b) Find the orthogonal trajectories of the family of curves $y = ax^2$.

(4+4+4+4)+4

2.(a) Solve the following differential equations:

(i) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$

(ii) $(D^2 + 5D + 6)y = e^{-2x} \sin 2x$

(iii) $(D^3 + D^2 - D - 1)y = \cos 2x$

(b) Solve $\frac{d^2y}{dy^2}$ $-2\frac{dy}{dx} = e^x \sin x$, using method of variation of parameters.

(5+5+5)+5

3.(a) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3 \sin 5t)$.

(b) Find $L^{-1}\left[\frac{s+1}{s^2+6s+25}\right]$

© Using convolution theorem, find $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$.

(d) Solve,using Laplace Transform, the differential equation

$(D+1)^2 y = t$ given that $y = -3$ when $t = 0$ and $y = -1$ when $t = 1$.

4+5+5+6

4.(a) Define ordinary and singular points of a homogeneous second order linear differential equation.

[Turn over

Find the power series solution of the equation

$$y'' + x y' + x^2 y = 0$$

in powers of x, about $x = 0$.

(b) Find series solution about $x = 0$ of the differential equation

$$x y'' + y' - y = 0$$

(2+8)+10

5 (a) Prove that

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0, \text{ if } \quad m \neq n$$

$$= \frac{2}{2n+1}, \text{ if } \quad m = n$$

(b) Prove that (i) $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$

(ii) $nP_n = x P_n' - P_{n-1}'$

(5+5)+(5+5)

6.(a) Expand the function $f(x) = x^2$ as Fourier series in $[-\pi, \pi]$. Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ... = \frac{\pi^2}{8}$$

(b) Expand $\pi x - x^2$ in the half range sine series in the interval $[0, \pi]$ up to three terms.

Deduce that $\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} + ... = \frac{\pi^3}{32}$

10+10

7(a) Solve the heat conduction equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$,

Given that $u(0,t) = 0 = u(l,t)$, and $u(x,0) = f(x)$, $0 \leq x \leq l$.

(b) A string is stretched and fastened to two points apart. Motion is started by displacing the string in the form of $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$y(x,t) = a \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}$

10+10