MCA 1st YEAR 2nd SEMESTER EXAM 2017

NUMERICAL METHODS

Time: 3 hours Full Marks: 100

Answer any **FIVE (5)** questions from the following

(All parts of the same question must be answered together)

1.

a. Evaluate f(1) using *Taylor's series* for $f(x) = x^3 - 3x^2 + 5x - 10$ (Hint: $x_i=0, x_{i+1}=1$)

b. Derive *Lagrange's Interpolation formula*. What is the disadvantage with this formula? Mention an *interpolation formula* that does not have this disadvantage.

c. Derive the *recurrence relation* using Chebyshev polynomial. Mention its application.

4+(7+2+1)+6=20

2.

a. Compute a real root of the equation $f(x) = e^{-x}$ by Newton-Raphson method. (Consider correctness upto 3 decimal places.)

b. Derive Newton's forward difference interpolation formula.
From the following table find the number of students who obtained marks between 60 and 70.

Marks obtained	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

Mention the method used by you.

8+(5+7)=20

3.

a. Explain the method of least squares to fit a straight line of the form $y = a_0 + a_1x$ to the data (x_i, y_i)

x	1	2	3	4	5	6
y	2.4	3.1	3.5	4.2	5.0	6.0

b. Deduce the general formula for *numerical integration*. Hence obtain *Simpson's 1/3-Rule* and use it to evaluate: $\int_3^7 x^2 \log x \, dx$

10+10=20

4.

a. Estimate the area (mention the rule used by you) bounded by the curve, x-axis, and extreme ordinates: (0,23), (0.5, 19), (1.0, 14), (1.5, 11), (2.0, 12.5), (2.5, 16), (3.0, 19), (3.5, 20), (4.0, 20)

Q 4 contd..

b. Find a real root between 2 and 3 of: $x \log_{10} x - 1.2 = 0$ using bisection method and secant method to a tolerance of 0.5%. Comment on the findings.

7+(12+1) = 20

5.

a. Factorize the matrix (given below) into the product LU where L and U have their usual meanings:

$$\begin{vmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{vmatrix}$$

b. Solve the following system of equations using Gauss elimination:

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 3z = 5$$

10+10=20

6.

a. Solve the following system of equations using Jacobi and Gauss-Seidel method:

$$4x + y - z = 3$$
$$2x + 7y + z = 19$$
$$x - 3y + 12z = 31$$

(Consider correctness upto 3 decimal places.)
Comment on the findings from both methods.

b. What are *eigenvalue* and *eigenvector* of a square matrix A? Hence derive the corresponding *characteristic equation* and *characteristic polynomial*. 15+5

7.

a. Given $dy / dx = 1/(x^2 + y)$ and $y(4) = 4$, obtain the Taylor series for $y(x)$ and compute $y(4.1)$.

b. Find the eigenvalues and eigenvectors of:

$$\begin{vmatrix} 1 & 3 & -3 \\ -3 & 7 & -3 \\ -6 & 6 & 2 \end{vmatrix}$$

c. Find d^2y/dx_2 at $x = 2$ for the following tabulated function and mention the formula/method used by you.

x	y
0	1
1	4
3	40
4	85

6+8+6=20