Ref:.No.:EX/PROD/MATH/T/124/2017(OLD)

B.Production Engineering 1st year 2nd Sem. Exam.2017(old).

MATHEMATICS – IVS (old)

Time: Three hours | Full Marks: 100

Answer any 10 questions.

1. State Dirichlet's condition for a Fourier series expansion of function. Find the Fourier series for the function $f(x)$ defined by

   $f(x) = x - x^2 \ ,-\pi \leq x \leq \pi$

   Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi}{12}$ 3+7

2. Determine Fourier Series expansion of the function

$$f(x) = 2 \ for \ 0 < x < \frac{2\pi}{3}$$
$$= 1 \ for \ \frac{2\pi}{3} < x < \frac{4\pi}{3}$$
$$= 0 \ for \ \frac{4\pi}{3} < x < 2\pi$$

10

3. Find the Fourier Sine series for

$$f(x) = \frac{1}{4} - x \quad for \ 0 < x < \frac{1}{2}$$
$$= x - \frac{3}{4} \quad for \quad \frac{1}{2} < x < 1$$

10

4. Obtain the half range Cosine series for

$$f(x) = x \ for \ 0 \leq x \leq \frac{l}{2}$$
$$= l - x \ for \ \frac{l}{2} \leq x \leq l$$

10

5. a) If $\vec{r} = (a\cos t)\vec{i} + (a\sin t)\vec{j} + (at\tan\alpha)\vec{k}$, then find $\left[\frac{d\vec{r}}{dt} \ \frac{d^2\vec{r}}{dt^2} \ \frac{d^3\vec{r}}{dt^3}\right]$ 6

   b) Show that $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\ ] = 2[\vec{a}\,\vec{b}\,\vec{c}]$ 4

6 Prove that $\bar{\nabla}.(\bar{A} \times \bar{B}) = \bar{B}.(\bar{\nabla} \times \bar{A}) - \bar{A}.(\bar{\nabla} \times \bar{B})$ 10

7. State Green's theorem. Apply Green's theorem to evaluate

   $\int_C [(y - Sinx)dx + Cosx\ dy]$,

   where c is the triangle enclosed by the lines y=0, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. 10

[Turn over

8. If $\bar{V} = 2xz\bar{i} + y^2\bar{j} + yz\bar{k}$, evaluate $\int_S \bar{V} \cdot \bar{n} dS$ where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$ 10

9 State Gauss Divergence theorem.
Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0$ and $x = a, y = a, z = a.$ 10

10. A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x, t)$. 10

11. A homogeneous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the centre is T and falls uniform to zero at the two ends. Find the temperature function $u(x, t)$. 10

12 A rectangular metal plate is bounded by the lines $x = 0, x = a,$ $y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is 10

13 A rectangular metal plate is bounded by the lines $x = 0, x = a,$ $y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is

$$u(x, y) = u_0 \operatorname{sech}\left(\frac{(b-a)\pi}{a}\right) \cosh\left(\frac{(b-y)\pi}{a}\right) \cos\left(\frac{\pi x}{a}\right)$$ 10