Ref:.No.:EX/PROD/MATH/T/124/2017(OLD)

B.Production Engineering 1st year 2nd Sem. Exam.2017(old).

MATHEMATICS – IV(old)

Time: Three hours Full Marks: 100

Answer any 10 questions.

1. State Dirichlet's condition for a Fourier series expansion of function. Find the Fourier series for the function $f(x)$ defined by

   $f(x) = x - x^2 \; , -\pi \le x \le \pi$

   Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi}{12}$ 3+7

2. Determine Fourier Series expansion of the function

   $$f(x) = 2 \;\; for \;\; 0 < x < \frac{2\pi}{3}$$
   $$= 1 \;\; for \;\; \frac{2\pi}{3} < x < \frac{4\pi}{3}$$
   $$= 0 \;\; for \;\; \frac{4\pi}{3} < x < 2\pi$$
   10

3. Find the Fourier Sine series for

   $$f(x) = \frac{1}{4} - x \;\; for \;\; 0 < x < \frac{1}{2}$$
   $$= x - \frac{3}{4} \;\; for \;\; \frac{1}{2} < x < 1$$
   10

4. Obtain the half range Cosine series for

   $$f(x) = x \;\; for \;\; 0 \le x \le \frac{l}{2}$$
   $$= l - x \;\; for \;\; \frac{l}{2} \le x \le l$$
   10

5. a) If $\vec{r} = (a\cos t)\,\vec{i} + (a\sin t)\,\vec{j} + (at\tan\alpha)\,\vec{k}$, then find 6

   $[\frac{d\vec{r}}{dt} \;\; \frac{d^2\vec{r}}{dt^2} \;\; \frac{d^3\vec{r}}{dt^3}]$

   b) Show that $[\,\vec{a} + \vec{b},\; \vec{b} + \vec{c},\; \vec{c} + \vec{a}\,] = 2[\,\vec{a}\,\vec{b}\,\vec{c}\,]$ 4

6 Prove that $\bar{\nabla}.(\bar{A} \times \bar{B}) = \bar{B}.(\bar{\nabla} \times \bar{A}) - \bar{A}.(\bar{\nabla} \times \bar{B})$ 10

7. State Green's theorem. Apply Green's theorem to evaluate

   $$\int_C [(y - Sinx)dx + Cosx\, dy],$$

   where c is the triangle enclosed by the lines y=0, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. 10

[Turn over

8. If $\bar{V} = 2xz\bar{\imath} + y^2\bar{\jmath} + yz\bar{k}$, evaluate $\int_S \bar{V}.\bar{n}dS$ where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$ 10

9 State Gauss Divergence theorem.
Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0, y = 0, z = 0$ and $x = a, y = a, z = a.$ 10

10. A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The midpoint is taken to a small height h and released from rest at time $t = 0$. Find the displacement function $y(x,t)$. 10

11. A homogeneous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the centre is T and falls uniform to zero at the two ends. Find the temperature function $u(x,t)$. 10

12 A rectangular metal plate is bounded by the lines $x = 0, x = a,\ y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is 10

13 A rectangular metal plate is bounded by the lines $x = 0, x = a,\ y = 0$, and $y = b$. The three sides $x = 0, x = a$, and $y = b$. are insulated and the side $y = 0$ is kept at temperatute $u_0 \cos(\frac{\pi x}{a})$. Show that the temperature in the steady state is

$$u(x,y) = u_0 \operatorname{sech}(\tfrac{(b-a)\pi}{a}) \cosh(\tfrac{(b-y)\pi}{a}) \cos(\tfrac{\pi x}{a})$$ 10