c) Prove that the series

$$\frac{2}{1}+\frac{3}{4}+\frac{4}{9}+\cdots+\frac{n+1}{n^2}+\cdots \text{ is divergent.}$$ 5+6+5

9. Apply Leibnitz-test to discuss convergence of the series

$$1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\cdots$$ 4

Bachelor of Engineering in Production Engineering Examination, 2017

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - III S

Time : Three hours Full Marks : 100

Symbols/Notations have their usual meanings.

Answer **Q.No. 9** and ***any six*** from the rest.

1. Find Laplace transform of the following functions :

   a) $e^{at}\sin bt$ b) $e^{3t}t^{5/2}$

   c) $e^{-3t}(2\cos 5t-3\sin 5t)$ d) $\frac{1-e^{-t}}{t}$. 4×4=16

2. a) Find the Laplace transform of the square-wave function of period a defined by

$$f(t)=\begin{cases}1, & 0<t<\frac{a}{2}\\ -1, & \frac{a}{2}<t<a\end{cases}.$$

   b) Let $L\{F(t)\}=f(p)$, then prove that

$$L\{F(t-a)u(t-a)\}=e^{ap}f(p).$$ 8+8

3. a) Evaluate inverse Laplace transform of

i) $\frac{1}{p^2-6p+10}$ ii) $\frac{3p+7}{p^3-2p-3}$.

b) Solve the differential equation

$$\frac{d^2y}{dt^2}+4y=4t$$

with $y(0)=1,\ y'(0)=5.$ 8+8

4. a) Find the Fourier transform of the function

$$f(x)=\begin{cases}1+\frac{x}{a}, & \text{for } -a<x<0\\ 1-\frac{x}{a}, & \text{for } 0<x<a\\ 0, & \text{elsewhere}\end{cases}$$

b) Evaluate the integrals, using Laplace transform :

$$\int_0^{\infty} e^{-3t}(t\sin t)dt$$ 10+6

5. a) Find the Fourier cosine transform of

$$f(x)=\begin{cases}\cos x, & 0<x<a\\ 0, & x\ge a\end{cases}.$$

b) Find the Fourier sine transform of

$$\frac{e^{-ax}}{x}.$$ 8+8

6. a) Solve the following different equations by finding proper integrating factors :

i) $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$

ii) $(2xy^4e^y+2xy^3+y)dx+(x^4y^4e^y-x^2y^2-3x)dy=0.$

b) Solve $(D^3+1)y=\text{Cos}x.$ 10+6

7. a) Use the method of variation of parameter to solve the differential equation

$$\frac{d^2y}{dx^2}+a^2y=\tan ax.$$

b) Solve $(D^2-1)y=xe^{2x}.$ 10+6

8. a) Is the series

$$\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\cdots$$

convergent or divergent ?

b) Prove that $\sum_{n=1}^{\infty}\frac{1}{n^2}<\infty.$