Ex/Prod/Math/T/114/2017(Old) (S)

BACHELOR OF ENGINEERING IN PRODUCTION ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

MATHEMATICS - IS (OLD)

Time : Three hours Full Marks : 100

Answer ***any 10*** questions.

1. a) If $y = x^n \log x$, find y_n.

   b) If $y = e^{a\text{Sin}^{-1}x}$, then prove that

   $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$ 4+6

2. a) State and prove Lagrange's Mean value theorem.

   b) If $y = x^{n-1}\log x$, show that $y_n = \frac{(n-1)!}{x}$ 6+4

3. a) Show that $\log(1+x) > x - \frac{x^2}{2}$, if $x > 0$

   b) Expand the function $\text{Sin}^3 x$ in a finite series with Lagrange's form of remainder. 4+6

4. a) Expand Cos hx in power of x in an infinite series.

   b) Show that $\text{Sec}x + \log \text{Cos}^2 x$ is a maximum for $x = 0$ and a minimum for $x = \pi/3$. 5+5

5. a) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$.

   b) Evaluate $\lim_{x\to 0}\frac{(e^x-1)\tan^2 x}{x^3}$. 6+4

6. a) If $u=\log(x^3+y^3+z^3-3xyz)$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=-\frac{3}{(x+y+z)^2}$$

   b) If $u=\text{Cos}^{-1}\{(x+y)/(\sqrt{x}+\sqrt{y})$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\text{Cot}\,u=0$$ 5+5

7. a) If $u=F(x^2+y^2+z^2)f(xy+yz+zx)$, prove that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

   b) If $u=x\phi(x+y)+y\psi(x+y)$, prove that

$$\frac{\partial^2 u}{\partial x^2}-2\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=0$$ 10+6

8. a) Show that the function x^2+xy+y^2-4x+y is a maximum at $(-7/3,-2)$

   b) Examine the existence of maxima or minima of the function $f(x,y)=xy$ subject to the condition $5x+y=13$. 5+5

9. Using Lagrange's method of undetermined multiplier; find the extreme value of $7x^2+y^2+8xy$, when $x^2+y^2=1$. 10

10. a) Given f(x) defined by

$$f(x)=\begin{cases}x^2 & \text{when} \quad 0\le x\le 1\\ \sqrt{x} & \text{for} \quad 1\le x\le 2\end{cases}.$$

   evaluate $\int_0^2 f(x)dx$.

   b) Evaluate $\int_0^2 |1-x|\,dx$. 5+5

11. a) Prove that $\frac{1}{2}<\int_0^1\frac{dx}{\sqrt{4-x^2}+x^3}<\frac{\pi}{6}$.

   b) If $n>1$, prove that

$$0\cdot 5<\int_0^{1/2}\frac{dx}{\sqrt{1-x^{2x}}}<0\cdot 524$$ 5+5

12. State and prove Par boux's theorem. 10