BACHELOR OF ENGINEERING IN PRODUCTION ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

MATHEMATICS - IIS (OLD)

Time : Three hours Full Marks : 100

(Notations/Symbols have their usual meanings)

Answer ***any ten*** questions.

1. a) If $\frac{1}{x+iy}+\frac{1}{u+iv}=1$; x, y, u, v being real, express v in terms of x and y.

   b) If $x+\frac{1}{x}=2\cos\theta$, show that

   $$\frac{x^{2n}+1}{x^{2n-1}+x}=\frac{\text{Cos}\,n\theta}{\text{Cos}(n-1)\theta}.$$

   c) Show that $\tanh^{-1}(\cos\theta)=\cosh^{-1}(\operatorname{cosec}\theta)$. 3+4+3

2. a) Prove that

   $$\frac{(\cos 5\theta-i\sin 5\theta)^2(\cos 7\theta+i\sin 7\theta)^{-3}}{(\cos 4\theta-i\sin 4\theta)^9(\cos\theta+i\sin\theta)^5}=1$$

   b) Prove that

   $$\text{Cos}^7\theta=\frac{1}{64}(\cos 7\theta+7\cos 5\theta+21\cos 3\theta+35\cos\theta).$$

[Turn over

c) If $z = x + iy$, find the real and imaginary parts of $\exp(z^2)$. 4+4+2

3. a) If $z = e^{i\theta}$, show that $\dfrac{z^2 - 1}{z^2 + 1} = i\tan\theta$.

b) If $u = \log\tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$, prove that

i) $\tanh\dfrac{u}{2} = \tan\dfrac{\theta}{2}$

ii) $\theta = i\log\tan\left(\dfrac{\pi}{4} + \dfrac{iu}{2}\right)$. 4+(3+3)

4. a) Prove, without expanding, that

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} \text{ vanishes.}$$

b) Factorize $\Delta = \begin{vmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{vmatrix}$ 5+5

5. a) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

11. a) Find the equation of the sphere through the points (2, 0, 1), (1, –5, –1), (0, –2, 3) and (4, –1, 2). Also find its centre and radius.

b) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 5+5

12. a) Show that the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should intersect orthogonally is

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}.$$

b) Find the radius of curvature at the point (3a/2, 3a/2) to the curve $x^3 + y^3 = 3axy$. 5+5

b) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

c) Solve the following equations by Cramer's rule :

$x + y + z = 3,\ \ x + 2y + 3z = 4,\ \ x + 4y + 9z = 6$ 3+2+5

6. a) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

b) Solve the following equations by matrix method :

$3x + 4y + 5z = 4,\ x + 2y = -1,\ 5x + y + z = 5$ 5+5

7. a) Show that the straight lines whose direction cosines are given by the equations.

$al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

b) Find the equation of the plane which passes through the points A(0, 1, 1), B(1, 1, 2) and C(–1, 2, –2).

6+4

8. a) A plane meets the coordinate axes at A, B, C such that the centroid of the triangle ABC is the point (a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$

b) Find in symmetrical form, the equations of the line

$x + y + z + 1 = 0,\ \ 4x + y - 2z + 2 = 0.$ 5+5

9. a) Show that the line $\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z-1}{2}$ is parallel to the plane $2x + 2y - z = 6,$ and find the distance between them.

b) Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are coplanar and find the equations of the plane containing them. 5+5

10. a) Find the shortest distance between the lines

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

b) Find the magnitude and equations of the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$ 4+6