Ex./PRN/MATH/T/221/2017

BACHELOR OF PRINTING ENGINEERING EXAMINATION, 2017

(2nd Year, 2nd Semester)

Mathematics - IV R

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use separate answer script for each part.

PART - I (50 marks)

Answer q.no.1 and any ***four*** from the rest.

1. Give an example of a sequence which is bounded but not convergent. 2

2. (a) A sequence $\{u_n\}$ is defined by $u_1 = \sqrt{2}$ and $u_{n+1} = \sqrt{2u_n}$ for $n \geq 1$.

   Prove that $\lim_{n\to\infty} u_n = 2$.

   (b) Prove that

   $$\lim_{n\to\infty} \frac{1+\sqrt{2}+\sqrt[3]{3}+....+\sqrt[n]{n}}{n} = 1$$ 6+6=12

(Turn over)

3. (a) Find the Fourier series expansion for f(x), if and only if $(x) = x^2, -1 < x < 1$.

(b) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. 6+6=12

4. (a) Define monotonic increasing sequence. Show that the sequence {f(n)} where

$$f(n) = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}, n \geq 1$$ is a monotonic increasing sequence.

(b) Prove that

$$\lim_{n \to \infty}\left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}}\right) = 1$$

6+6=12

5. (a) Test the convergence of the series

$$\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + ...$$

(b) Test the convergence of the series

$$\frac{1}{1.2^2} + \frac{1}{2.3^2} + \frac{1}{3.4^2} + ...$$ 6+6=12

10. (a) Let, $f = x^3 + y^3 + z^3$ be a scalar point function. Find

(i) The directional derivative of f at (1,–1,2) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$. 4

(ii) Find the direction in which the directional derivatives is maximum at (1,–1,2). 2

(iii) Find the maximum directional derivative at (1,–1,2). 2

(b) Determine the constant 'a' so that, the vector $\vec{F} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solonoidal. 2

(c) Find constants a,b,c such that the vector

$\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ becomes irrotational. 4

(d) Find the unit vector normal to the level surface given by $\phi(x,y,z) = x^2 + y - z$ at the point (1,0,0). 2

11. (a) Show that, $\vec{\nabla}^2\left(\frac{1}{r}\right) = 0$, where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. 6

(b) Prove that, if $\vec{\nabla}^2\phi = 0$, then $\vec{\nabla}\phi$ is both solenoidal and irrotational, where ϕ has continuous 2nd order partial derivatives. 4

(Turn over)

(b) Find the scalar product of two vectors given by two diagonals of a unit cube. What is the angle between them? 4

(c) Prove by vector method, that the median to the base of an isosceles triangle is perpendicular to the base. 2

(d) Show that,

$$\left(\vec{a} \times \vec{b}\right)^2 = |\vec{a}|^2 , |\vec{b}|^2 - \left(\vec{a} . \vec{b}\right)^2 \qquad 2$$

9. (a) Show that

$$\left[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 \qquad 4$$

(b) Find the projection of the vector $\vec{A} = 4\hat{i} - 3\hat{j} + \hat{k}$ on the line passing through the points (2,3,–1) and (–2,–4,3). 4

(c) A particle, acted on by constant forces $\vec{F}_1 = 4\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{F}_2 = 3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point A(1,2,3) to the point B(5,4,1). Find the work done by the forces on the particle. 4

(d) Find the magnitude of the moment about the point A(3,–1,3) of a force vector $4\hat{i} + 2\hat{j} + \hat{k}$ passing through B(5,2,4). 4

6. (a) State D'Alembert's ratio test and Cauchy's root test.

(b) Test the convergence of the series

$$1 + \frac{1}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \ldots \qquad 6+6=12$$

7. (a) Test the convergence of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$$ by Leibnritz test.

(b) State the Sandwich theorem for sequence. Find the convergence and divergence behaviour of the sequence $\{r^n\}$ for different real values of r. 6+6=12

PART - II (50 marks)

Answer ***q.no. 13*** and any ***three*** from the rest.

8. (a) In the triangle ABC on a plane, let $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$ Prove by vectors that

$$\text{Cos}\,A = \frac{b^2 + c^2 - a^2}{2bc} \quad \text{and} \quad \frac{\text{Sin}\,A}{a} = \frac{\text{Sin}\,B}{b} = \frac{\text{Sin}\,C}{c}$$

where, $a = |\vec{a}|$, $b = |\vec{b}|$ and $c = |\vec{c}|$. 4+4=8

(Turn over)

(c) Suppose, $\vec{A} = \left(3x^2 + 6y\right)\hat{i} - (14yz)\hat{j} + (20xz^2)\hat{k}$

Evaluate, $\int_c \vec{A}\,.\,d\vec{r}$ from (0,0,0) to (1,1,1) along the path given by the straight lines joining (0,0,0) to (1,0,0) to (1,1,0) to (1,1,1). 6

12. (a) State Stoke's theorem. 3

(b) Verify Green's theorem in the plane for

$\oint_c \left[(y - \sin x)dx + \cos x\, dy\right]$ where C is the triangle with vertices $A\left(\frac{\pi}{2}, 0\right)$, $B\left(\frac{\pi}{2}, 1\right)$ and 0(0,0). 6

(c) Let, $\vec{F} = (4xz)\hat{i} - \left(y^2\right)\hat{j} + (yz)\hat{k}$.

Evaluate $\iint_S \left(\vec{F}\,.\,\hat{n}\right)ds$ where, S is the surface of the cube bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$ and $\hat{n}$ is the outward drawn unit normal to the surface. 7

13. Find the volume of the parallelopiped with edges represented by the vectors,

$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + 2\hat{j} - \hat{k}, \vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$ 2

—— X ——