BACHELOR OF PRINTING ENGINEERING EXAMINATION, 2017

(2nd Year, 1st Semester, Supplementary)

MATHEMATICS

Paper - IIIR

Time : Three Hours　　　　Full Marks : 100

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer **any five** questions.

1. Solve the following differential equations:

   (a) $(x^2 - 3y^2)dx + 2xydy = 0$

   (b) $(x^2 + 1)\dfrac{dy}{dx} + 4xy = x$

   (c) $(x^2 + y^2)^{-7}dx + xdx + ydy = 0$

$7+7+6 = 20$

2. Solve the following differential equations:

   (a) $(D^2 - 2D - 3)y = 2e^x - 10\sin x$

   (b) $(D^2 - 3D + 2)y = 2x^2 + e^x + 2xe^x + 4e^{3x}$

   (c) $(D^2 + 1)y = x\sin x$

$8+6+6 = 20$

3. (a) Use the Method of Variation of Parameters to solve the following differential equation:

   $$(D^2 + 1)y = \tan x$$

   (b) Solve

   $$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$$

(c) Find the Particular Integral of the following differential equation:

$$(D^2 + 4D + 5)y = e^{-2x}(1 + \cos x)$$

8+6+6 = 20

4. (a) Find the Laplace transform of the following functions:

   (i) $te^{-t} \sin 3t$

   (ii) $(\sin t - \cos t)^2$

   (b) Find the inverse Laplace transform of the following functions:

   (i) $\dfrac{p}{p^2 - 6p + 13}$

   (ii) $\dfrac{12 - 5p}{p^2 + 4}$

   (c) Use Laplace transform technique to solve the following differential equation:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$$

   subject to the conditions $y(0) = 0$, $y'(0) = 0$.

6+6+8 = 20

5. (a) Show that the series solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0,$$

   can be put in the form

$$y = A\cos x + B\sin x,$$

   where A and B are two arbitrary constants.

   (b) Show that

$$P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$$

   is a solution of the Legendre equation

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$

10+10 = 20

6. (a) Eliminate the arbitrary function f to form a partial differential equation:

$$f(xy + z^2, x + y + z) = 0.$$

(b) Solve the following partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + z = 0$$

subject to the conditions: $\dfrac{\partial z}{\partial x} = 1$ and $z = e^y$ when $x = 0$.

(c) Use the method of separation of variables to solve the following partial differential equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

subject to the condition $u(x, 0) = 6e^{-3x}$.

7+7+6=20

7 (a) Use Lagrange's method to solve the following partial differential equation

$$p \tan x + q \tan y = \tan z \ , \ p = \frac{\partial z}{\partial x} \ , \ q = \frac{\partial z}{\partial y}.$$

(b) Use Charpit's method to solve the following partial differential equation

$$px + qy = pq, \ p = \frac{\partial z}{\partial x} \ , \ q = \frac{\partial z}{\partial y}.$$

(c) Classify the following partial differential equations:

$$(i) \quad u_{xx} - u_{tt} = 0, \ (ii) \ u_{xx} + 4u_{xy} + 4u_{yy} = 0, \ (iii) \ 2u_{xx} + 4u_{xy} + 3u_{yy} = 2.$$

6+8+6=20

8. Find the non-trivial solution of the one dimensional wave equation

$$c^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0, \quad 0 \le x \le l \ , \ t > 0,$$

subject to the conditions: $u(0, t) = u(l, t) = 0$, $u_t(x, 0) = 0$, $u(x, 0) = f(x)$.

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9. Find the non-trivial solution of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \le x \le 10 \ , \ y \ge 0,$$

subject to the conditions: $u(0, y) = 0$, $u(10, y) = 0$, $u(x, \infty) = 0$, $u(x, 0) = f(x)$.

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