7. (a) Find the extreme values of $f(x, y) = x^3 - 3xy + y^3$.

   (b) Find the length of the curve $y = \log \sec x$ between $x = 0$ and $x = \pi/3$.

   (c) Evaluate

$$\iint_R \operatorname{Sin}(x+y)\,dx\,dy \text{ over } R : \{0 \le x \le \pi/2\ ;\ 0 \le y \le \pi/2\}$$

8. (a) Find the volume of solid formed by rotation of an elupse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about minor axis.

   (b) Show that area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $16a^2/3$.

   (c) Evaluate $\int_0^4 e^x\,dx$ taking 4 subintervals by

   (i) Simpson's one third rule

   (ii) Trapezoidal rule.

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Ex./PRN/MATH/T/112/2017(S)

BACHELOR OF PRINTING ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - I R

Time : Three hours — Full Marks : 100

Answer any ***five*** questions.

All questions carry equal marks.

1. (a) Prove that $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

   (b) Show that $\underset{x \to 0}{\mathrm{Lt}} \sin 1/x$ does not exist.

   (c) Find the values of a and b such that

$$f(x) = \begin{cases} 3ax + b, & x < 0 \\ 5 + 3\operatorname{Sin} x, & x > 0 \end{cases}$$

   is differentiable at $x = 0$.

   (d) Find $\frac{d^n y}{dx^n}$ where $y = \log x$.

2. (a) State Rolle's Theorem. If $\varphi(x)$ is a polynomial and λ is real, then there exists a root of $\varphi'(x) + \lambda\varphi(x) = 0$ between any pair of roots of $\varphi(x) = 0$.

(Turn over)

(b) Prove that $\frac{x}{1+x} < \ln(1+x) < x, \forall\, x > 0$.

(c) Show that $f(x) = x^3 - 6x^2 + 12x + 50$ has neither a maximum nor a minimum at $x = 2$.

3. (a) If $\underset{x\to 0}{\text{Lt}} \frac{\text{Sin}\, 2x + a\,\text{Sin}\, x}{x^3}$: be finite then find the value of a and the limit.

(b) Find $\underset{x\to 0}{\text{Lt}} \left(\frac{\tan x}{x}\right)^{1/x}$.

(c) State Taylor's theorem.

(d) Expand $e^{\sin x}$ in Taylor's series as far as term containing x^3.

4. (a) Evaluate

(i) $\int_0^1 x^3 \left(1 - x^2\right)^{5/2} dx$

(ii) $\int_0^{\pi/2} \text{Sin}^4 x\, \text{Cos}^4 x\, dx$

(iii) $\int_0^{\pi/2} \sqrt{\tan x}\, dx$

(b) Show that $\int_0^{\pi/2} \text{Cos}^4 x\, dx = \frac{3\pi}{16}$.

5. (a) Evaluate $\underset{y\to 0}{\text{Lt}}\ \underset{x\to 0}{\text{Lt}}\ f(x)$ and $\underset{x\to 0}{\text{Lt}}\ \underset{y\to 0}{\text{Lt}}\ f(x)$ where $f(x) = \frac{x+y}{x-y}$. Does the $\underset{\substack{x\to 0\\ y\to 0}}{\text{Lt}} f(x)$ exists ? Justify.

(b) If $u(x, t) = e^{-t}\, \text{Sin}\, x$, find the value of $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2}$.

(c) If $z = y\, \text{Cos}\, xy$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

6. (a) If $u = \text{Sin}^{-1}\left(\frac{x+y}{x-y}\right)$ then find the value of $x^2 u_{xx} + 2xy\, u_{xy} + y^2\, u_{yy} = 0$.

(b) If $u = (ax + by)^2 - (x^2 + y^2)$ where $a^2 + b^2 = 2$ show that $u_{xx} + u_{yy} = 0$.

(c) If $u = \text{Sin}^{-1} \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(Turn over)