BACHELOR OF POWER ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - II Q

Time : Three hours Full Marks : 100

Notations / Symbols have their usual meanings.

Answer any ***ten*** questions.

1. (a) Find the complex number z, such that $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=\frac{2\pi}{3}$.

(b) If $x+\frac{1}{x}=2\cos\theta$, then find $x^r+\frac{1}{x^r}$.

(c) Simplify

$$\frac{(\cos 5\theta - i\sin 5\theta)^2(\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9(\cos\theta + i\sin\theta)^5}$$

3+4+3

2. (a) Show that

$$\cos^7\theta=\frac{1}{16}(\cos 7\theta+7\cos 5\theta+21\cos 3\theta+35\cos\theta)$$

(Turn over)

(b) If $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, prove that

$$\theta = -i \log\tan\left(\frac{\pi}{4} + \frac{iu}{2}\right)$$

(c) If $z = a^{i\theta}$, show that $\frac{z^2 - 1}{z^2 + 1} = i\tan\theta$. 3+4+3

3. (a) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

(b) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and I is the unit matrix of order 3, then evaluate $A^2 - 3A + 9I$. 5+5

4. (a) If $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ in which a, b, c are different, then show that abc = 1.

10. Obtain the various possible solutions of the one-dimensional wave equation

$$u_{tt} = c^2 u_{xx}$$

by the method of separation of variables. Which one of these solutions is appropriate with the physical nature of the equation ? Justify your answer. 10

11. Solve the equation

$$u_t = u_{xx}$$

subject to the conditions $u(x,0) = 3 \sin nn\, x$, $u(0,t) = 0$ and $u(1,t) = 0$, where $0 < x < 1$, $t > 0$. 10

12. Find the general solutions of the PDEs :

(a) $(z - y)p + (x - z)q = y - x$

(b) $xp - yq = y^2 - x^2$

(c) $y^2p - xyq = x(z - 2y)$ 3+3+4

—— X ——

7. Find the Fourier series expansion of the function $f(x) = x^2, -\pi \leq x \leq \pi$.

Hence show that

(a) $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$

(b) $\sum \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(c) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$ 6+4

8. Express the function $f(x) = x$ as a half range sine and cosine series in $0 < x < 2$. 5+5

9. (a) Form a PDE by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.

(b) Form a PDE by eliminating the arbitrary function f from $z = f\left(\frac{xy}{z}\right)$.

(c) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. 3+3+4

(b) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

(c) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$$

4+3+3

5. (a) Solve the following equations by Cramer's rule :

$x + y + z = 4,\ x - y + z = 0,\ 2x + y + z = 5.$

(b) Solve the following equations by matrix method :

$3x + 4y + 5z = 4,\ x + 2y = -1,\ 5x + y + z = 5$ 5+5

6. (a) Test for convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + ...\infty$$

(b) Discuss the convergence of the series

(i) $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$

(ii) $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + ...\infty$ 4+(3+3)

(Turn over)