11. (a) Find $\int_0^1 \frac{dx}{x^2}$ if possible. What is the Cauchy value of this integral.

(b) Find the entire length of the curve $r = a \cos 2\theta$, within $\theta = 0$ to $\theta = \pi$.

12. (a) Prove that the greatest rectangle inscribed in a given circle is a square.

(b) Find the value of $\int_0^\infty \frac{dx}{1+x^2}$, if possible.

(Symbols have their usual meanings)

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Ex./PE/MATH/T/112/2017(S)

BACHELOR OF POWER ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - I Q

Time : Three hours | Full Marks : 100

Answer any ***ten*** questions.

All question carries equal marks.

1. (a) Prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

(b) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 10$, $|\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 6$, find $|\vec{a} \times \vec{b}|$. Also find unit normal vector to the plane of vectors $\vec{a}$ and $\vec{b}$.

2. (a) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$ under what condition $(\vec{a} \times \vec{b}) \times \vec{c} = a \times (\vec{b} \times \vec{c})$.

(b) Examine whether the vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$, and $\hat{i} + 2\hat{j} + 3\hat{k}$ are coplaner or not ?

3. (a) The components of a contravariant vector in (x^i) coordinate system are 5 and 6. Find its components in $(\bar{x}^i)$ coordinate system, if $\bar{x}^1 = 3x^1 + 4x^2$, $\bar{x}^2 = 5x^1 - 2x^2$.

(b) Prove that dx^i is a contravariant vector and δ^i_j is a mixed tensor of type (1,1).

4. (a) Evaluate $\int_0^a \int_0^a \int_0^a x^2 \, dxdydz$.

(b) Prove that $B(m,n) = B(n,m)$ and also find $\Gamma\left(\frac{1}{2}\right)$.

5. (a) Prove that $\Gamma(n+1) = \underline{|n}$, for positive integer n.

(b) Evaluate $\int_0^1 x^4 (1-x)^7 \, dx$.

Using Beta-Gamma functions.

6. (a) State Rolle's theorem and verify it for the function $f(x) = |x|$, $-1 \le x \le 1$.

(b) Find $\lim_{x \to \pi/2} (\sin x)^{\tan x}$.

7. (a) Find the value of θ in the Mean value theorem $f(a+h) = f(a) + h f'(a+\theta h)$, $0 < \theta < 1$ for the function $f(x) = ax^2 + bx + c$.

(b) Find Taylor series expansion for the function $f(x) = e^x$ about $x = 0$.

8. (a) If $\cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

(b) If $y = (x^2-1)^n$, then show that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

9. (a) Evaluate $\int_0^{\pi/2} \int_0^{\gamma/2} \sin(x+y)\,dxdy$.

(b) If $y = x^{2n}$, where n is a positive integer, then show that $y_n = 2^n\{1.3.5....(2n-1)\}x^n$.

10. (a) Prove that $\left[\vec{\alpha}+\vec{\beta} \quad \vec{\alpha}+\vec{\gamma} \quad \vec{\gamma}+\vec{\alpha}\right] = 2\left(\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}\right)$.

(b) Given $x + y = 3$; find the maximum and minimum values of $\frac{9}{x} + \frac{36}{y}$.