10. Solve by method of separation of variable :

$$c^2 u_{xx} - u_{tt} = 0,\ \ 0 \le x \le a,\ t \ge 0,$$

subject to $u(0,t) = u(a,t) = 0$ and $u(x,0) = f(x)$, $u_t(x,0) = g(x)$. 10

11. (a) Solve : $(D^2+9)y = x \text{ Sin } x$

(b) $x\cos x\dfrac{dy}{dx} + y(x\,\text{Sin}\,x + \text{Cos}\,x) = 1$ 5+5

12. Define Bersel's function of order n and show that $J_{+n}(x)$ and $J_{-n}(x)$ are linearly dependent.

Also prove that $\displaystyle\int_0^1 \frac{u J_0(xu)}{(1-u^2)^{1/2}}du = \frac{\text{Sin}\,x}{x}$ 5+5

—— X ——

Ex./PE/MATH/T/122/2017

BACHELOR OF POWER ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester)

Mathematics - III Q

Time : Three hours Full Marks : 100

Answer any ***ten*** questions.

1. (a) Find the velocity and acceleration of a particle which moves along the curve $x(t) = 2\sin 3t$, $y(t) = 2\cos 3t$, $z = 8t$ at $t = \dfrac{\pi}{12}$. Also find the magnitude of the velocity and acceleration at that point.

(b) Find the curvature and torsion of a space curve $\vec{r}(t) = a\cos t\ \hat{i} + a\sin t\ \hat{j} + bt\hat{k}$ at any point $t > 0$. 4+6

2. (a) Prove that $\vec{P} = e^{-ax}\left(\vec{A}\,\text{Sin}\,ay + \vec{B}\,\text{Cos}\,ay\right)$ satisfies $\dfrac{\partial^2 \vec{p}}{\partial x^2} + \dfrac{\partial^2 \vec{p}}{\partial y^2} = 0$, where $\vec{A}$ and $\vec{B}$ are constant vectors and 'a' is a scalar.

(b) Find the unit normal vector and first fundamental form for a surface $\vec{r} = (u+v)\hat{i} + (u-v)\hat{j} + 4uv\hat{k}$. 5+5

3. (a) Find the directional derivative of $\psi = 2x^2z - 3xy^2z^2$ at (3,1,–3) in the direction $3\hat{i} - 4\hat{j} + 5\hat{k}$.

(b) Find 'a' for which $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational. 5+5

4. (a) State Gaus's divergence theorem. Use it to evaluate $\iint_S \left\{\left(x^3 - yz\right)\hat{i} - 2x^2y\,\hat{j} + 2\hat{k}\right\}.\vec{n}\,ds$, where S denotes the surface of the cube bounded by the planes $x=0$, $x=a$; $y=0$, $y=a$; $z=0$, $z=a$.

(b) If $\overrightarrow{f(t)} = (t^2 - t)\hat{i} + (3t^4 - 4)\hat{j} + 5t\hat{k}$, find $\int_2^3 \overrightarrow{f(t)}\,dt$. 7+3

5. (a) Show that the straight lines whose direction cosines are given by $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.

(b) Find the equation of the plane through (2,5,–8) and perpendicular to each of the planes $2x - 3y + 4z + 1 = 0$ and $4x + y - 2z + 6 = 0$. 5+5

6. (a) Find the magnitude of the shortest distance between the st. lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$.

(b) Prove that the st. lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar and also find the equation of the plane in which they lie. 5+5

7. (a) Find the centre and diameter of the circle where the plane $x - 2y + 2z = 3$ intersects the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$.

(b) Solve $y = px + 4/p$, where $p = \frac{dy}{dx}$ and obtain its singular solution. 5+5

8. (a) Solve :

$y(1+xy)dx + x(1-xy)dy = 0.$

(b) Solve by method of variation of parameters for $y_2 + a^2y = \tan ax$. 4+6

9. (a) Define Legendre polynomial and show that

$$n\,P_n(x) = x\,P_n'(x) - P_{n-1}'(x)$$

(b) Prove that

$$\int_{-1}^{1} P_n(x)\,dx = 0, \text{ if } n \geq 1.$$ 6+4

(Turn over)