Bachelor of Engineering in Metallurgical Engineering Examination, 2017

(2nd Year, 1st Semester, Supplementary)

Mathematics - IIIN

Time : Three hours Full Marks : 100

Answer ***Question No. 9*** and ***any six*** questions from the rest.

(Notations have their usual meanings)

1. Solve the following differential equation :

   a) $y(2xy + e^x)dx - e^y dy = 0$

   b) $(1 + y^2)dx - (\tan^{-1} y - x)dy = 0$

   c) $xydx + (2x^2 + 3y^2 - 20)dy = 0$ 5+6+5

2. a) Solve the differential equation :

   $(D^3 - D^2 + 3D + 5)y = e^x \mathrm{Cos}2x$

   b) Solve by the method of variation of parameters

   $(D^3 - 6D^2 + 11D - 6)y = e^{3x}.$ 8+8

3. a) Solve the differential equation :

   $(D^3 + 2D^2 + D)y = x^2 + x$

   b) Solve by the method of variation of parameters :

   $(D^2 - 3D + 2)y = \dfrac{e^x}{1 + e^x}$ 8+8

[Turn over

4. a) Let $f(t) = \frac{t}{a}$ for $0 < t < a$

$= 1$ for $t > 1$

Find the Laplace transform of $f(t)$.

b) Find the inverse Laplace transform of

$$\frac{S+7}{S^2+2S+5}$$

c) Use Convolution theorem to find

$$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$$

5+5+6

5. a) If $Z = x + iy$ and x, y are real, find the locus of 'Z' if $\frac{z+i}{z+2}$ is real.

b) If $\text{Cos}\alpha + \text{Cos}\beta + \text{Cos}\gamma = \text{Sin}\alpha + \text{Sin}\beta + \text{Sin}\gamma = 0$ show that $\text{Cos}(\alpha+\beta) + \text{Cos}(\beta+\gamma) + \text{Cos}(\gamma+\alpha) =$

$$\text{Sin}(\alpha+\beta) + \text{Sin}(\beta+\gamma) + \text{Sin}(\gamma+\alpha) = 0$$

c) Prove that

$$\text{Sin}7\theta = 7\text{Sin}\theta - 56\text{Sin}^2\theta + 112\text{Sin}^5\theta - 64\text{Sin}^7\theta.$$

5+5+6

6. a) Prove that

$$\text{Cos}^6\theta + \text{Sin}^6\theta = \frac{1}{8}(3\text{Cos}4\theta + 5).$$

b) Prove that $\tan(\alpha + i\beta) = x + iy$, show that

$$x^2 + y^2 + 2x\,\text{Cot}2\alpha = 1$$

c) Find the general value of i^i. 5+6+5

7. a) Find the series solution near x = 0 of the equation

$$x^2\frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x-9)y = 0$$

b) Prove that $J_{n+1}(x) = \frac{2x}{x}J_n(x) - J_{n-1}(x)$ 10+6

8. a) Prove that $\int_{-1}^{1} P_m(x)P_n(x) = 0$, when $m \neq n$

$= \frac{2}{2n+1}$, when $m = n$

b) Prove that $(2x+1)P_n(x) = P_{n+1}'(x) - P_{n-1}'(x)$ 10+6

9. Prove that

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x).$$

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