Ref. No. Ex/MET/MATH/T/113/2017(s)

BACHELOR OF ENGINEERING IN METALLURGICAL ENGINEERING SUPPLEMENTARY EXAMINATION, 2017

1^{st} year, 1^{st} semester
Mathematics-IN

Time: 3 hours **Full Marks: 100**

(Symbols have their usual meaning)

Answer ***any ten*** from the following questions.

1. a) Prove by induction that for each $n \in \mathrm{N}$, and for each $n \geq 2$, $(n+1)! > 2^n$.
b) Show that the function f, defined by $f(x) = x\sin\frac{1}{x}$, when $x \neq 0$, approaches to 0 as $x \to 0$.

[5+5]

2. a) State Sandwich theorem for limit of a function and hence prove that $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$.
b) Prove that $\lim_{x\to 0}\frac{e^x-1}{x} = 1$. [6+4]

3. a) Examine the continuity of the following function at (0,0)

$$f(x,y) = \begin{cases} \frac{xy\,(x^2-y^2)}{x^2+y^2}, & \text{where } x^2+y^2 \neq 0 \\ 0 &, \text{where } x^2+y^2 = 0. \end{cases}$$

b) Establish $\lim_{\substack{x\to 0 \\ y\to 0}} xy\frac{x^2-y^2}{x^2+y^2} = 0$, using $\varepsilon - \delta$ definition. [5+5]

4. a) Verify that $\log\tan x$ $(0 < x < \frac{\pi}{2})$ has neither a maximum nor a minimum at $x = 2$.
b) Investigate the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ -1, & x = 0 \end{cases}.$$

[5+5]

5. a) Use M.V.T to show that $\frac{x}{1+x} < \log(1+x) < x$, if $x > 0$.
b) State Mean Value Theorem (Langrange's form) and give the geometrical interpretation of this theorem.

[5+5]

6. a) If $y = \frac{1}{x^2+a^2}$, then find y_n.
b) Establish $\lim_{x\to\infty}\frac{2x^8-7x^2+5}{9x^4+5x+12} = \infty$. [6+4]

7. a) State Taylor's infinite series and hence expand $f(x) = \sin x$ in powers of x.
   b) Show that x^x has a minimum value at $x = \frac{1}{e}$. [5+5]

8. (a) Find the value of $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{1^2+n^2}+\frac{n}{2^2+n^2}+\cdots+\frac{n}{(n-1)^2+n^2}\right]$.

   (b) Evaluate $\int_0^{\pi/2}\frac{x}{\sin x+\cos x}dx$.

[5+5]

9. (a) Examine the convergence of improper integral $\int_0^{\infty}\frac{\cos x}{1+x}dx$.

   (b) Test the convergence of $\int_1^{\infty}\frac{dx}{x\sqrt{1+x^2}}$.

[5+5]

10. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$; $m > 0$, $n > 0$.

    (b) Evaluate $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta$. [5+5]

11. (a) Evaluate $\iint y\,dx\,dy$ over the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

    (b) Find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Hence, find the area enclosed by the given ellipse.

[5+5]

12. (a) Define bounded sequence. Find the lower and upper bounds of the sequence $\{x_n\}$, where $x_n = \frac{2n-7}{3n+2}$.

    (b) Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{n!}{(n^n)^2}$. [5+5]

13. (a) Test the convergence of the series

    $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\cdots+\frac{2^n-2}{2^n+1}x^{n-1}+\cdots$

    (b) Show that the p-series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ converges for $p > 1$ and diverges for $p \geq 1$. [6+4]