13. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then verify that

i) A satisfies of its own characteristic equation.

ii) Hence find A^9 and

iii) Find also A^{-1}

iv) Find the matrix represented by

$$2A^5 - 3A^4 + 2A^3 - A^2 + I.$$

14. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, show that for every integer

$n \geq 3, \quad A^n = A^{n-2} + A^2 - I.$ Hence determine A^{50}.

Ex/MET/Math/T/122/2017

BACHELOR OF ENGINEERING IN METALLURGICAL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester)

MATHEMATICS - IIN

Time : Three hours　　　　Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

(Notation/Symbols have their usual meanings)

PART - I

Answer ***any five*** questions.

1. a) If two vectors are parallel, then show that one of them can be expressed as a scalar multiple of the other.

   b) Show that the three points $\hat{i} - 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $-7\hat{j} + 10\hat{k}$ are collinear.

   c) Show, by vector method, that the straight line joining the mid points of any two sides of a triangle is parallel to the third side and half of its length. 3+3+4

2. a) Show that the necessary and sufficient condition for two vectors $\vec{a}$ and $\vec{b}$ are perpendicular is $\vec{a} \cdot \vec{b} = 0$.

   b) Show that, if $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are mutually perpendicular.

   c) Prove, by vector method, that the parallelograms on the same base and between the same parallels are aqual in area. 3+3+4

[Turn over

3. a) If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, then show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

b) Given two vectors $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, express $\vec{b}$ in the form $\vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$. 5+5

4. a) If $\vec{u}(t)$ and $\vec{v}(t)$ are two vector functions of the scalar variable t, then show that

$$\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \frac{d\vec{u}}{dt}\cdot\vec{v} + \vec{u}\cdot\frac{d\vec{v}}{dt}.$$

Hence show that

$$u\frac{du}{dt} = \vec{u}\cdot\frac{d\vec{u}}{dt}, \text{ where } u = |\vec{u}|.$$

b) Show that if $\vec{r} = \vec{a}\sin\omega t + \vec{b}\cos\omega t$, where $\vec{a}\cdot\vec{b}$ are constant vectors and ω is a constant scalar, then

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r} \quad \text{and} \quad \vec{r}\times\frac{d\vec{r}}{dt} = -\omega\,\vec{a}\times\vec{b}.$$ 6+4

5. a) Find the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at the point (2, –1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

b) If n be a positive integer and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then show that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$.

11. a) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then find A^2 and show that $A^2 – A^{-1}$.

b) Show that for the skew-symmetric matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$$

$(I–A)(I+A)^{-1}$ is an orthogonal matrix.

12. a) Reduce the following matrix to the normal form and find its rank

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

b) Find the eigen values of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

PART - II

Answer ***any five*** questions.

8. a) Prove that

$$\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$$

b) Solve by Cramer's rule of the system of equations

$$\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 14 \\ x-y+z &= 2 \end{aligned}$$

9. a) Prove that if A and B be an orthogonal matrices of the same order then AB is orthogonal.

b) Solve by matrix method the system of equations

$$\begin{aligned} x+z &= 0 \\ 3x+4y+5z &= 2 \\ 2x+3y+4z &= 1 \end{aligned}$$

10. a) Show that $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$ is divisible by λ^2 and find the other factors.

b) If $u = x^2yz$ and $v = xy - 3z^2$, then find

i) $\nabla(\nabla u \cdot \nabla v)$ ii) $\nabla \cdot (\nabla u \times \nabla v)$.

c) Define a solenoidal vector. 3+5+2

6. a) If $\vec{f} = 3xy\hat{i} - y^2\hat{j}$ evaluate $\int_C \vec{f}.d\vec{r}$, where C is the curve in the xy-plane $y = 2x^2$ from (0, 0) to (1, 2).

b) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\int_c (x\,dy - y\,dx)$. Hence find the area of an ellipsa whose semi-major and semi-minor axes are of lengths a and b. 3+7

7. a) State the Stoke's theorem.

b) Define an irrotational vector.

c) A fluid motion is given by

$$\vec{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}.$$

Is this motion is irrotational ? If so, find the velocity potential. 2+2+6