Ref. No.: Ex/ME/T/326H/2017

INTRODUCTION TO FINITE ELEMENT METHOD FOR MECHANICAL ENGINEERS

B.E. MECHANICAL ENGINEERING
THIRD YEAR SECOND SEMESTER EXAM 2017

Answer any five questions Time: 3 hours

Question 1

a) Derive the stiffness matrix of a bar element using minimization of potential energy.

b) Write down the transformation matrix of a two-dimensional bar element and show the process of determining the stiffness matrix in global coordinates.

c) What transformation matrix should you use for a three-dimensional bar element?

Question 2

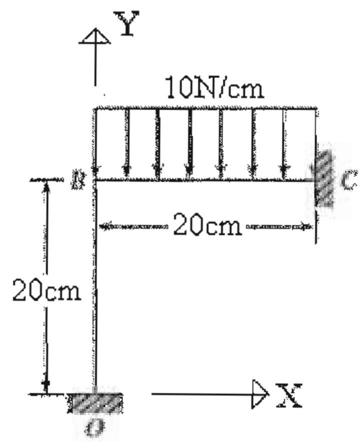

Figure Q2

Find the equivalent nodal loads for the uniformly distributed load shown in Figure Q2

For both the elements b=h=2cm. The modulus of elasticity is $2\times 10^7\ N/cm^2$

The expression for element stiffness matrix is

$$\underline{k}=\frac{E}{L}\times$$

$$\begin{bmatrix} AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2+\frac{12I}{L^2}S^2\right) & -\left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2+\frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A-\frac{12I}{L^2}\right)CS & -\left(AS^2+\frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2+\frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix}$$

Derive the final three simultaneous equations after incorporation of boundary conditions. Refer Figure Q2

Question 3

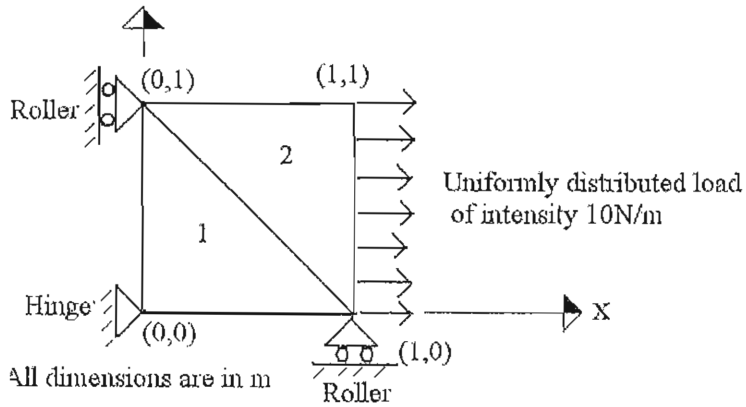

Figure Q3

An assembly of two constant-strain triangles is shown in Figure Q3. Assume **plane stress** conditions. Take thickness as $t = 0.001m$. **All dimensions are in meters**.

For the sake of calculation take $\frac{E}{1-\mu^2} = 200GPa$ and $\mu = 0.25$

Use the relation $N_i = \frac{1}{2\Delta}(a_i + b_i x + c_i y)$

Where, $a_1 = x_2 y_3 - x_3 y_2 \quad b_1 = y_2 - y_3 \quad c_1 = x_3 - x_2$

a) How many degrees of freedom does this system have after elimination of the boundary conditions?
b) Assemble the element stiffness and the force vector only for the effective (free) degrees of freedom

Question 4

a) Mention all the stress terms in cylindrical co-ordinates for an axisymmetric problem
b) Draw a triangular 3 node axisymmetric element and show the nodal degrees of freedom.
   Although the shape function is identical to a CST, yet the stress is not constant throughout the element. Explain.
c) Derive the expression for stiffness matrix for such an element?

Question 5

(a) Derive the shape functions for a 4-node quadrilateral isoparametric finite element

(b) Sketch the shape functions

(c) Describe the process of forming the stiffness matrix for this element

(d) What is Jacobian of transformation?

Question 6

a) Write down the shape functions of a nine-node isoparametric quadrilateral using Lagrange interpolation function

b) Sketch the above shape functions.

c) Evaluate the integral $\int_{-1}^{1}\int_{-1}^{1} r^2 s^2 dr ds$. Use 2 point and 3 point Gauss quadrature rule. Use the data given in Table 1. Are the results same? Explain your answer.

Table 1. Data for 2 point and 3 point Gauss quadrature rule

Number of points	Locations	Weights
2	$\pm$0.57735 02691 89626	1.00000 00000 00000
3	$\pm$0.77459 66692 41483	0.55555 55555 55556
	0.00000 00000 00000	0.88888 88888 88889