Ref. No.: Ex/ME/T/223/2017

BME 2nd Year 2nd Semester EXAMINATION, 2017

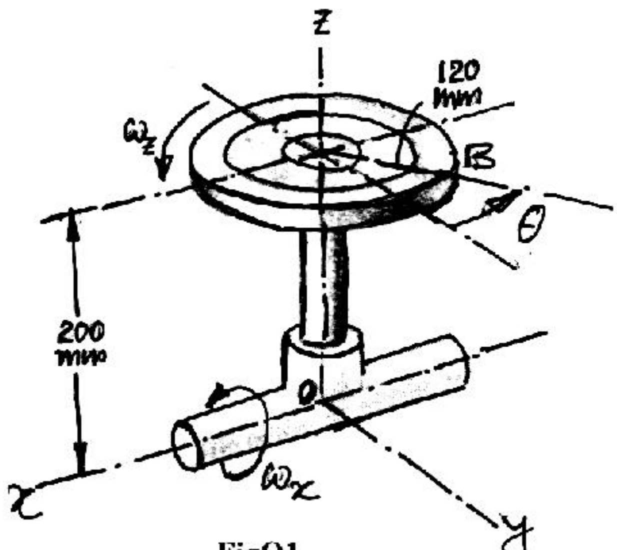
Engineering Mechanics IV

Time: Three (3) hours **Full marks: 100**

GROUP A (Dynamics of Rigid Bodies)

Answer any Three (3) Questions **3×16=48**

Q1. Refer **FigQ1**. The Circular disc of **120 mm** radius rotates about the z-axis at the constant rate ω_z =**20 rad/s**, and the entire assembly rotates about the fixed x- axis at the constant rate ω_x = **10 rad/s**. Calculate the magnitude of the velocity V and the acceleration a of point B for the instant when $\theta = 0$.



FigQ1

Q2. Deduce **Euler's equation** (as **given in the Appendix**) stating all necessary assumptions.

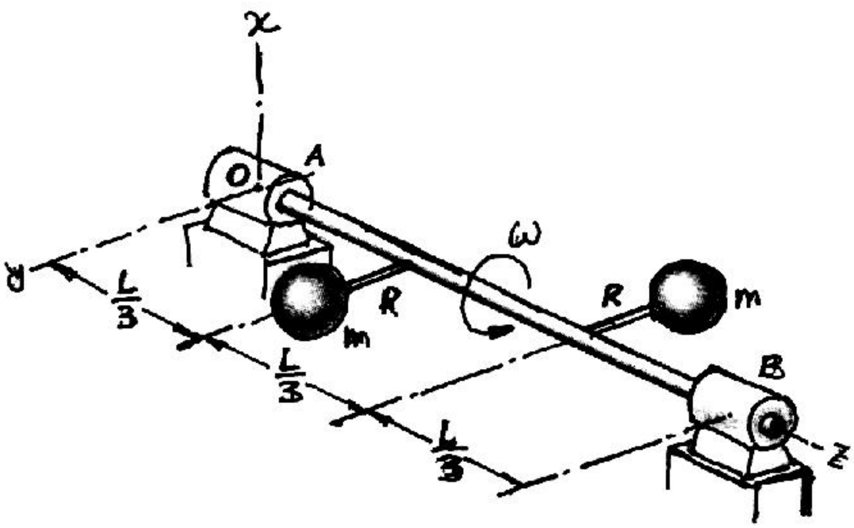
Q3. Refer **FigQ1** again. Find the **moment at the joint O** due to the indicated motion using –

(i) Euler's Equation (as shown in appendix)

(ii) Euler's Equation in terms of Euler's angle (as shown in appendix)

Q4. Refer **FigQ4 (shown in the next page)**. The slender shaft carries two offset particles each of mass m and rotates about the z-axis with the constant angular rate ω as indicated. Determine the x and y components of the **bearing reactions at A** and **B** due to the **dynamic imbalance** of the shaft for the position shown using **equations of rigid body motion**.

[Turn over



FigQ4

Q5.(i) The rotational transformation of the mass moment of inertia matrix for a rigid body follows the rule –

$[I'] = [R][I][R]^T$. Here $[R]$ is the transformation matrix for axis rotation. Prove this relation.

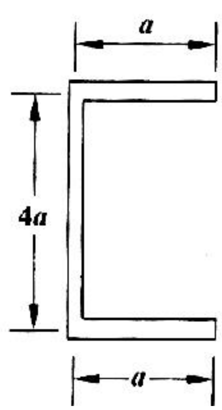
(ii) The kinetic energy of a rigid body in space motion is given by –

$T = \frac{1}{2} m v_c^2 + \frac{1}{2}\{\omega\}^T [I]\{\omega\}$. Prove this relation. Symbols carry usual meaning.

Group-B (Advanced Strength of Materials)

Answer any Three (3) Questions 3×16=48

Q6. Refer to **FigQ6** showing the cross-section of a thin-walled beam with uniform wall thickness ($t \ll a$). Locate the shear centre of the section.



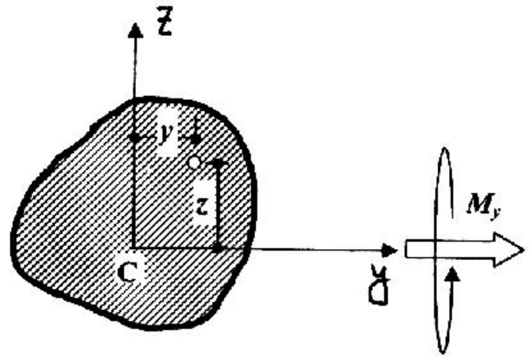
FigQ6

Q7. What do you mean by unsymmetric bending of beams?

FigQ7 shows the cross-section of a beam with bending moments acting in the directions as given in the figure. From first principles, derive the following equation of the bending stress at a point (*y*,*z*) in the cross-section:

$$\sigma_x = \frac{M_y\left(-yI_{yz} + zI_z\right)}{\left(I_y I_z - I_{yz}^2\right)}$$

Write the corresponding bending stress equation when *y*-*z* axes are the principal axes of inertia through the centroid.



FigQ7

Q8. Derive the equation for radial and circumferential stresses in a thick-walled cylinder which is subjected to uniform external pressure intensity p_0 acting on it.

Where the stresses will be maximum?

Show the variations of the stresses along the radial distance of the cylinder.

Q9. Write the stress equilibrium equations in three dimensional rectangular co-ordinate system explaining each term of the equations clearly. Hence derive the results.

Consider in a deformable body the stress field (in **MPa**) at any point (*x*,*y*,*z*) is given by:

$$\sigma_{xx} = \left(3x^2 - 3y^2 - z\right); \quad \sigma_{yy} = 3y^2; \quad \sigma_{zz} = \left(3x + y - z\right);$$

$$\sigma_{xy} = \left(z - 6xy\right); \quad \sigma_{yz} = 0; \quad \sigma_{xz} = \left(x + y\right);$$

Determine the body forces (if any) required to satisfy equilibrium condition.

Q10. When in a beam **curved beam theory** is to be applied? From first principles, derive the Winkler's stress equation for a curved beam:

$$\sigma = \frac{M(R_n - R)}{A e R}$$

In the equation R_n is the radius of curvature of the neutral line and $e = R_n - \bar{R}$ where $\bar{R}$ is the centroidal radial distance. Write the basic assumptions required to be made to derive the above equation.

Appendix

Euler' equations:

$$\Sigma M_x = I_x \dot{\omega}_x - \omega_y \omega_z \left(I_y - I_z\right)$$

$$\Sigma M_y = I_y \dot{\omega}_y - \omega_z \omega_x \left(I_z - I_x\right)$$

$$\Sigma M_z = I_z \dot{\omega}_z - \omega_x \omega_y \left(I_x - I_y\right)$$

Euler' equations in terms of Euler angles (Equations of Gyrodynamics):

$$\Sigma M_x = I'\left(\ddot{\theta} - \dot{\psi}^2 \sin\theta \cos\theta\right) + I\dot{\psi}\left(\dot{\psi}\cos\theta + \dot{\phi}\right)\sin\theta$$

$$\Sigma M_y = I'\left(\ddot{\psi}\sin\theta + 2\dot{\psi}\dot{\theta}\cos\theta\right) - I\dot{\theta}\left(\dot{\psi}\cos\theta + \dot{\phi}\right)$$

$$\Sigma M_z = I\frac{d}{dt}\left(\dot{\psi}\cos\theta + \dot{\phi}\right)$$

Here ψ = Angle of precession, θ = Angle of nutation and ϕ = Angle of spin