Ex/ME/Math/T/221/2017

Bachelor of Engineering in Mechanical Engineering Examination, 2017

(2nd Year, 2nd Semester)

Mathematics IV

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer any ***five*** questions

1. a) Define subspace of a vector space V over a field F. Prove that $W_1 = \{(a_1, a_2, \cdots, a_n) \in \mathbb{R}^n$ such that $a_1 + a_2 + \cdots + a_n = 0\}$ is a subspace of $\mathbb{R}^n$ but $W_2 = \{(a_1, a_2, \cdots, a_n) \in \mathbb{R}^n /$ $a_1 + a_2 + \cdots + a_n = 1\}$ is not.

   b) Prove that intersection of two subspaces of a Vector space is a subspace. Is it tone for union ? Justify. 6+4

2. a) Prove that a linearly independent set of vectors in a finite dimensional vector space V over a field F is either a basis of V or can be extended to a basis of V.

   b) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, when

   $$W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0,\ 2x + y + 3z = 0\}$$ 6+4

3. a) Prove that $|<\alpha, \beta>| \le \|\alpha\|\,\|\beta\|$. Hence prove that $\|\alpha + \beta\| \le \|\alpha\| + \|\beta\|$.

   b) If $\beta_1 = (3,0,4)$, $\beta_2 = (-1,0,7)$ and $\beta_3 = (2,9,11)$ are in $\mathbb{R}^3$ equipped with the standard inner product, use Gram-Schmidt process to obtain orthogonal basis of $\mathbb{R}^3$ from $\beta_1, \beta_2, \beta_3$. 5+5

4. a) Determine the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1) (1, 1, 0) of $\mathbb{R}^3$ to the vectors (2, 0, 0), (0, 2, 0), (0, 0, 2) respectively. Find kerT and ImT.

   b) If V and W are two vector spaces over the same field F and V is finite dimensional then prove that nullity of T + rank of T=dim V, where $T : V \to W$ is a linear transformation. 5+5

[Turn over

5. a) Define an inner product space. Show that for an inner product space (V, <, >), for $x, y, z \in V$, $c \in F$.

   i) $<x, y+z> = <x, y> + <x, z>$

   ii) $<x, 0> = <0, x> = 0$

   iii) $<x, x> = 0 <=> x = 0$.

   b) Let a mapping T be defined on $\mathbb{R}^2$ to $\mathbb{R}^2$ by T(x,y) = (x+2, y). Check whether it is a linear or not ? 2+3

   c) Find the eigen values of the given matrix

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

6. a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by T(x, y, z) = (x + y, 2z–x). If {(1, 0, –1), (1, 1, 1), (1, 0, 0)} and {(0, 1), (1, 0)} are ordered bases of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively, find the matrix of T relative to the above bases.

   b) Check whether the vectors (3, 2, 1), (0, 1, 2) and (1, 0, 2) are basis of $\mathbb{R}^3$ or not ?

Ref. No.: EX/ME/MATH/T/221/2017

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION,2017

(2nd Year, 2nd Semester)

MATHEMATICS-IV

Time: Three hours | Full Marks:100

(50 marks for each part)
Use separate answer script for each part.

Part-II

Answer any five questions.

1. Write short notes on i) random experiment ii) sample space iii) compound event iv) certain event and v) impossible event (10)

2. (a) Give frequency definition of probability. (2)

   (b) Use the above definition to find the expression for conditional probability of an event B on the hypothesis that the event A has occured. (2)

   (c) Given three events A,B and C such that $P(A \cap B \cap C) \neq 0$ and $P(C|A \cap B) = P(C|B)$, show that $P(A|B \cap C) = P(A|B)$. (2)

   (d) A balanced die is tossed twice. If A is the event that an even number comes up on the first toss, B is the event that an even number comes up on the second toss, and C is the event that both tosses result in the same number. are the events A, B, and C (i) pairwise independent ii) independent? (4)

3. An urn contains four balls numbered 1,2,3, and 4. If two balls are drawn (without replacement) from the urn at random(that is, each pair has the same chance of being selected) and Z is the sum of numbers on the two balls drawn, find

   (a) the probability distribution of Z and draw a histogram (5)

   (b) the distribution function of Z and draw its graph (5)

4. (a) Write down the necessary conditions for which an arbitrary function $f(x)$ can serve as probability density function of a continuous random variable X. (2)

   (b) The probability density function of a continuous random variable X is given by (6)

$$f(x) = \begin{cases} \frac{c}{\sqrt{x}}, & 0 < x < 4 \\ 0, & otherwise \end{cases}$$

   Find (i) the value of c (ii) $P(X < \frac{1}{4})$ and $P(X > 1)$

   (c) Find the distribution function of the above random variable. (2)

[Turn over

5. (a) Find $E[X], E[X^2]$ and $E[(X - E[X])^2]$ for the random variable X which has the probability density (6)

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & elsewhere \end{cases}$$

(b) Given the joint probability density (4)

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases}$$

find the marginal densities of X and Y

6. (a) The joint and marginal distribution of two discrete random variables X and Y are given below. (6)

		x			
		0	1	2	$h(y)$
	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{7}{12}$
y	1	$\frac{2}{9}$	$\frac{1}{6}$	0	$\frac{7}{18}$
	2	$\frac{1}{36}$	0	0	$\frac{1}{36}$
	$g(x)$	$\frac{5}{12}$	$\frac{1}{2}$	$\frac{1}{12}$	1

Find covariance of X and Y.

(b) Prove that the correlation coefficient between two random variables $\rho(X, Y)$ satisties the relation (4)

$$-1 \leq \rho(X, Y) \leq 1$$

7. (a) If the regression of Y on X is linear, then (5)

$$\mu_{Y|X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$

(b) The following data pertain to the chlorine residual in a swimming pool at various times after it has been treated with chemicals: (5)

Number of hours	*Chlorine residual (parts per million)*
2	1.8
4	1.5
6	1.4
8	1.1
10	1.1
12	0.9

(i) Fit a least square line from which we can predict the chlorine residual in terms of the number of hours since the pool has been treated with chemical.
(ii) Use the equation of least squares line to estimate the chlorine residual in the pool 5 hours after it has been treated with chemicals. (Write your answer correct upto 4 decimal places)