BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2017

(2nd Year, 1st Semester, Supplementary)

MATHEMATICS - III

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART I

(Answer any five questions.)

1. (i) Find $L^{-1}\left\{\frac{1-3s}{s^2+8s+21}\right\}$. 3+3+4=10

   (ii) Find $L^{-1}\left\{\frac{3s-2}{2s^2-6s-2}\right\}$.

   (iii) Show that $L\{\cos at\} = \frac{s}{s^2+a^2}$, using the definition of Laplace Transform.

2. (i) Using Laplace transform. find solution of $y'' + 25y = 10\cos(5t)$, where $y(0) = 2, y'(0) = 0$.

   (ii) Find $L^{-1}\left\{\frac{s-2}{(s-2)^2+25} + \frac{s+4}{(s+4)^2+81} + \frac{1}{(s+2)^2+9}\right\}$. 6+4=10

3. (i) State and prove the first shifting theorem of Laplace Transformation.

   (ii) Find the complex Fourier Transform of $f(x) = e^{-a|x|}$, where x belongs to $(-\infty, \infty)$.

   6+4=10

4. (i) Find the Fourier series expansion for f(x), if f(x) = |x|, $-\pi < x < \pi$.

   (ii) Find the Fourier series expansion for f(x), if f(x) = x^2, $-\pi < x < \pi$.

   5+5=10

5. (i) Find the Fourier series for $1 + x$ on [-π,π].

   (ii) Find the Fourier transform of $e^{-4(x-3)^2}$. 5+5=10

[Turn over

6. (i) Find the Fourier cosine transform of e^{-x^2} and hence evaluate Fourier sine transform of xe^{-x^2}.

(ii) Solve $\frac{\partial u}{\partial t} = 2\,\partial u^2/\partial x^2$, if $u(0,t) = 0, u(x,0) = e^{-x}\ (x > 0),\ u(x,t)$ is bounded where $x > 0, t > 0$.

5+5=10

7. Show that $L\{\cos^2 t\} = \frac{s^2+2}{s(s^2+4)}$ using $L\{f'(t)\} = sF(s) - f(0)$, where $L\{f(t)\} = F(t)$.

10

PART II

GROUP - A

Answer ***any three*** questions. 3R10=30

8. a) Solve : $(5x^4+3x^2y^2-2xy^3)dx+(2x^3y-3x^2y^2-5y^4)dy$

   b) Solve : $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$ 4+6

9. a) Solve : $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$

   b) Solve the equation, by the method of variation of parameters $\frac{d^2y}{dx^2}+n^2y=\sec nx$ 6+4

10. a) Find the series solution of the equation $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$

    abour the point x = 0.

    b) Show that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\,\text{Sinx}$ 7+3

11. a) Show that, $(2n+1)P_n(x)-P_{n+1}(x)-P'_{n-1}(x)$

    b) Solve : $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$ 5+5

12. a) Solve : $\frac{d^2y}{dx^2}-\frac{2dy}{dx}+y=xe^x\sin x$

    b) Show that, $\frac{d}{dx}\left[x^4J_n(x)\right]=x^4J_{n-1}(x)$ 5+5

GROUP - B

Answer ***any two*** questions. $2\times10=20$

13. a) Find the partial differential equation by the elimination of a, b from $z=(x+a)(y+b)$

    b) Solve : $(y^2+z^2-x^2)p-2xyq+2zx=0$ 4+6

14. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x(l-x)$, find the displacement of the string at any distance x from one end at any time t. 10

15. Determine the solution of the one dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$, where the boundary conditions are $u(0,t)=0$, $u(l,t)=0 (t>0)$ and the initial condition $u(x,0)=x$, l being the length of the bar. 10