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Ref. No. ME/Math/T/111/2017(S)

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING
SUPPLEMENTARY EXAMINATION, 2017

(1st year 1st Semester)

Mathematics – I

Time: Three hours

Full Marks: 100

(50 marks for each part)
Use separate Answer script for each part.

PART – I

Symbols & Notations have their usual meanings.
Answer any ***FIVE*** questions.

1. (a) If $y = \cos(m \sin^{-1}x)$, then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

(b) Verify Lagrange's mean value theorem for the function

$$f(x) = x(x-1)(x-2) \ \ in \ [0, \tfrac{1}{2}].$$ [5+5]

2. (a) State Cauchy's Mean Value Theorem.

(b) Does $f(x) = |x|$ satisfy Rolle's theorem on $[-1,1]$? Justify your answer.

(c) Use Mean Value Theorem to show that $x \le \sin^{-1} x < \frac{x}{\sqrt{(1-x^2)}}$. [2+3+5]

3. (a) Use Maclaurin's theorem to expand $f(x) = \sin x$ in infinite series and give the range of validity of expansion.

(b) Use L'Hopital's rule to evaluate $\lim_{x\to 0} \log_{\tan^2 x}(\tan^2 2x)$.

[5+5]

4. (a) If $\rho_1 and\ \rho_2$ be the radii of curvature at the ends of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that $\rho_1^{\frac{2}{3}} + \rho_2^{\frac{2}{3}} = (a^2 + b^2)(ab)^{-\frac{2}{3}}$.

(b) Find the equation of the circle of curvature of $2xy + x + y = 4$ at the point $(1,1)$.

[5+5]

5. (a) Show that $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$ is continuous at (0,0) .

(b) Test the differentiability of the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & x^2 + y\text{^}2 \neq 0 \\ 0 & , x^2 + y^2 = 0 \end{cases}$

at (0,0) .

[5+5]

6. (a) State and prove Euler's Theorem for a homogeneous function in two variables x and y of degree n.

(b) If $= \tan^{-1}\frac{x^3+y^3}{x-y}$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u\,(1 - 4\sin^2 u).$$

[5+5]

7. (a) Define Limit of a Sequence and hence show that $\{x_n\} \to \frac{1}{2}$ as $n \to \infty$, where $x_n = \frac{n^2+1}{2n^2+3}$.

(b) Examine the convergence of the series $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots \quad (x \geq 0)$.

[5+5]

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(50 marks for each Group)

(Symbols and notations have their usual meanings)

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GROUP-B (50 Marks)

Answer *Q. No. 8* and any *three* from the rest.

8. Evaluate $\int_0^{\frac{\pi}{2}} \sin x \log(\sin x)\ dx$. 5

9. a) A function f defined on $[0,1]$ by $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

   Find $\underline{\int_0^1} f\,dx$ and $\overline{\int_0^1} f\,dx$.

   Examine whether f is Riemann integral in $[a,b]$. 8

   b) Let f:[-3, 3]→ R be define by $f(x) = \begin{cases} 2x\sin\frac{\pi}{x} - \pi\cos\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

   Examine whether f is Riemann integral in [-3, 3] and hence find $\int_{-3}^{3} f\,dx$. 7

10. a) Evaluate $\iint_D \sqrt{4a^2 - x^2 - y^2}\ dxdy$, where region D is the upper half of the circle $x^2 + y^2 - 2ax = 0$. 8

    b) Show that $\int_0^{\frac{\pi}{2}} e^{-\sin x}\,dx < \frac{\pi}{2}(1 - e^{-1})$. 7

11. a) Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2}}$, the field of integration being the positive octant of the sphere $x^2 + y^2 + z^2 = 1$. 7

    b) For the function f defined as $f(x,y) = \begin{cases} \frac{1}{y^2} & \text{if } 0 < x < y < 1 \\ -\frac{1}{x^2} & \text{if } 0 < y < x < 1 \\ 0 & \text{otherwise if } 0 \le x,\ y \le 1 \end{cases}$

    Does the integral of $\iint_R f(x,y)\,dx\,dy$ exists over R=[0, 1; 0, 1]? 8

12. Examine the convergence of the following improper integrals:

    a) $\int_0^1 \frac{dx}{(x+1)(x+2)\sqrt{x(1-x)}}$ b) $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\,dx$ c) $\int_0^\infty \frac{\sin x}{x}\,dx$ 15

13. a) State and proved fundamental theorem for integral calculus. 7

    b) Calculate the value of $\int_0^1 (4x - x^2)\ dx$ using (i) Trapezoidal rule, (ii) Simpson's $\frac{1}{3}$ rule by taking 10 intervals. Compute the exact value and find the absolute error in your result. 8
