subject to the following boundary and initial conditions :

$$u(0,t)=u(L,t)=0, \quad t\geq 0$$

$$u(x,0)=f(x)=\begin{cases} \frac{2k}{L}x, & 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x), & \frac{1}{2}<x<L \end{cases}$$

and $u_t(x,0)=0$. 10

13. Apply the method of separation of variables to obtain a formal solution of one dimensional heat equation

$$\frac{\partial u}{\partial t}=K\frac{\partial^2 u}{\partial x^2}, \quad 0\leq x\leq L$$

along with the boundary conditions

$u(0,t)=0,\ u(L,t)=0;\ t\geq 0$ and the initial condition $u(x,0)=f(x)$. 10

14. Solve the Laplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0;\ 0\leq x\leq a,\ 0\leq y\leq b$$

satisfying the boundary conditions :

$$u(0,y)=0,\ u(x,0)=0,$$

$$u(x,b)=0,\ \frac{\partial u}{\partial x}(a,y)=\sin^3\left(\frac{\pi y}{a}\right)$$

by applying the method of separation of variables. 10

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester, Old Syllabus)

MATHEMATICS - IVM

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions. 5×10=50

1. What do you mean by Fourier series of a function $f:[-\pi,\pi]\to\mathbb{R}$? Show that the Fourier series of the function $f(x)=\sin x+\cos 2x,\ -\pi\leq x\leq\pi$ is the function itself. 10

2. State Fundamental Theorem of convergence of Fourier series. Find the Fourier series for the function defined by

$$f(x)=\begin{cases} -\pi \text{ for } -\pi<x<0 \\ x \text{ for } 0<x<\pi \end{cases}$$

Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty=\frac{\pi^2}{8}$. 10

3. Find the Fourier series of the function

$$f(x)=\begin{cases} \cos x, & 0\leq x\leq\pi \\ -\cos x, & -\pi\leq x\leq 0 \end{cases}$$

Hence deduce that $\frac{2}{1.3}-\frac{6}{5.7}+\frac{10}{9.11}-\cdots\infty=\frac{\pi}{4\sqrt{2}}$. 10

4. Find the Fourier series of the function $f(x) = x\sin x, -\pi \le x \le \pi.$ Hence deduce that $\frac{1}{2}+\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots\alpha=\frac{\pi}{4}.$ 10

5. a) Find the half range Fourier cosine series of the function $f(x) = e^x, 0 < x < 1.$ 5

   b) Find the half range sine series of the function $f(x) = x(\pi - x)$ in $0 < x < \pi.$ 5

6. Find the Fourier series of the function $f(x) = x - x^2,\ -\pi < x < \pi.$ Hence deduce that

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\infty=\frac{\pi^2}{12}.$$ 10

7. State Parseval's Identity. Using this identity show that $\sum_{n=1}^{\alpha}\frac{1}{n^4}=\frac{\pi^4}{90}$ by Calculating the Fourier series of the function $f(x) = x^2, -\pi < x < \pi.$ 10

PART - II

(Symbols have their usual meaning)

Answer ***any five*** of the following questions.

8. a) Form the partial differential equation by eliminating the function ϕ from $\phi\left(x+y+z,\ \ x^2+y^2-z^2\right)=0.$

   b) Describe Lagrange's method to solve a partial differential equation. 5+5

9. a) Find the general integral of $z^2 - pz + qz + (x+y)^2 = 0$

   b) Use Lagrange's method to solve the p.d.e,

   $p\cos(x+y) + q\sin(x+y) = z.$ 5+5

10. a) Use Charpit's method to solve the equation $px + qy = pq.$

    b) Find the general integral of the partial differential equation $p^2 + q^2 = n^2.$ 5+5

11. a) Find the complete integral and singular integral of the equation $z = px + qy - 2\sqrt{pq}.$

    b) Solve : $\left(x^2+y^2\right)\left(p^2+q^2\right)=1.$ 5+5

12. Find the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\ ;\quad 0<x<L.$$