Bachelor of Engineering in Mechanical Engineering Examination, 2017

(1st Year, 1st Semester, Old)

Mathematics - IM (Old)

Time : Three hours — Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions.

Symbols/Notations have their usual meanings.

1. a) If $y = e^{ax}\sin bx$ then show that $y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax}\sin\left(bx + n\tan^{-1}\frac{b}{a}\right)$.

   b) If $I_n = D^n(x^n \log x)$ then prove that $I_n = n!\left[\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+\cdots+\log x\right]$. 5+5

2. a) State and prove Lagrange's Mean value theorem.

   b) Examine applicability of Rolle's theorem in [–1, 1] for the function f(x) = |x|. 6+4

3. a) Prove that $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1, \ \forall x > 0.$

   b) Show that Lagrange's remainder after n terms tends to zero when n is large and hence find the infinite series for e^x. 4+6

4. a) Determine the values of a and b so that

$$\lim_{x\to 0}\frac{a\sin 2x - b\sin x}{x^3} = 1.$$

   b) If $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & \text{when} \quad x^2+y^2 \neq 0 \\ 0 & \text{when} \quad x^2+y^2 = 0 \end{cases}$

   then show that $f_x(0,0) = 1$ and $f_y(0,0) = -1$. 5+5

[Turn over

5. a) If H(x,y) be a homogeneous function of x and y of degree n having continuous partial derivatives and $u(x,y)=(x^2+y^2)^{-\frac{n}{2}}$, show that $\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(H\frac{\partial u}{\partial y}\right)=0$

b) If $u=(x^2+y^2+z^2)^{-\frac{1}{2}}$ then prove that

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=u$ and $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0.$ 5+5

6. a) State and prove Euler's theorem of homogeneous functions of two variables.

b) Find the points on the curve $y=x^2+3x+4$ the tangent at which passes through the origin. 5+5

7. a) Find the radius of curvature at any point t for the ellipse $x=a\cos t,\ \ y=b\sin t$.

b) Show that $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & \text{when } x^2+y^2\neq 0\\ 0, & \text{when } x^2+y^2=0\end{cases}$

is not continuous at (0, 0). 5+5

ME/Math/T/112/2017(OLD) (S)

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics-IM (OLD)

Time: Three hours | Full Marks: 100

(50 marks for each part)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each part

PART-II (50 Marks)

Answer *Q. No. 8* and *any three* from the rest.

8. Evaluate $\int_0^1 x^3 (1-x^2)^{5/2}\, dx$. 5

9. a) Prove that $\int_0^1 \left\{\int_0^1 \frac{x-y}{(x+y)^3}\, dy\right\} dx \neq \int_0^1 \left\{\int_0^1 \frac{x-y}{(x+y)^3}\, dx\right\} dy$

   Does the integral of $\iint_R \frac{x-y}{(x+y)^3}\, dx\, dy$ exists over R=[0, 1; 0, 1]? 8

   Justify your answer.

   b) Changing the order of integration, evaluate $\int_0^\infty \int_0^\infty e^{-xy} \sin nx\, dxdy$.

   Hence deduce that $\int_0^\infty \frac{\sin nx}{x}\, dx = \frac{\pi}{2}$. 7

10. a) A function f defined on $[a, b]$ by $f(x) = x^2$. Find $\int_{\underline{a}}^{b} f\, dx$ and $\int_a^{\bar{b}} f\, dx$.

    Examine whether f is Riemann integral in $[a, b]$. 8

    b) Show that $\frac{\pi^2}{9} < \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{\sin x}\, dx < \frac{2\pi^2}{9}$. 7

11. a) Use first mean value theorem to show that

    $\frac{\pi}{6} \leq \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{(1-x^2)(1-\theta^2 x^2)}} \leq \frac{\pi}{6} \frac{1}{\sqrt{1-\frac{\theta^2}{4}}}, \quad \theta^2 < 1.$ 8

    b) Evaluate $\iint_D x^2 y\, dxdy$ over the first quadrant of the ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$. 7

12. Examine the convergence of the following improper integrals:

    a) $\int_0^2 \frac{1}{\sqrt{x(2-x)}}\, dx$ b) $\int_0^1 \frac{dx}{(x+1)(x+2)\sqrt{x(1-x)}}$ c) $\int_0^\infty \frac{\sin x}{x}\, dx$ 15

13. a) Let f:[-a, a]→ R be define by $f(x) = \begin{cases} 3x^2 \cos\frac{\pi}{x^2} + 2\pi \sin\frac{\pi}{x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

    Examine whether f is Riemann integral in [-a, a] and hence find $\int_{-a}^{a} f\, dx$. 7

    b) Evaluate $\iiint xyz\, dxdydz$, the field of integration being the positive octant of the ellipsoid $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} \leq 1$. 8

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