Bachelor of Engineering in Mechanical Engineering Examination, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - IIM (Old)

Time : Three hours | Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions.

All questions carry equal marks.

1. a) Evaluate the determinant

$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix}$$

b) Show that the skew-symmetric determinant of order 4 is a perfect square. 5+5

2. a) Given the system of equations :

$$x_1 + 4x_2 + 2x_3 = 1$$

$$2x_1 + 7x_2 + 5x_3 = 2k$$

$$4x_1 + mx_2 + 10x_3 = 2k + 1,$$

[Turn over

find for what values of k and m. The system has (i) a unique solution, (ii) no solution, (iii) many solutions.

b) Show that a real square matrix can be expressed as a sum of symmetric and skew-symmetric matrices uniquely.

6+4

3. a) State and prove Cayley-Hamilton theorem.

b) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, show that $A^3 = A^{-1}$. 5+5

4. a) Apply matrix inversion method to find the solution of a system of equations

$$2x - 2y - 4z = 8$$
$$2x + 3y + 2z = 8$$
$$-x + y - z = \frac{7}{2}$$

b) Determine the rank of the following matrix :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 2 \\ 1 & 3 & 5 & 7 & 4 \\ 1 & 4 & 7 & 10 & 8 \end{bmatrix}$$

6+4

12. a) Find the eigen-values and the corresponding eigen-functions of the equation

$$\frac{d^2y}{dx^2} + \lambda y = 0, y'(0) = 0, y'\left(\frac{\pi}{2}\right) = 0$$

b) Find a series solution of the equation

$$(x^2 - 1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0, \quad y(0) = 4, \quad y'^{(0)} = 6$$

5+5=10

13. a) Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$

b) Define ordinary and singular points of the differential equation $P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$ 5+5=10

PART - II

Answer question no. ***13*** and ***any four*** from the rest.

8. a) Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}\cos x$

   b) Solve : $\frac{d^2y}{dx^2} + 4y = x^2 \sin x$ 5+5=10

9. a) Solve : $y(2xy + e^x)dx - e^x dy = 0$

   b) Let M, N have continuous first partial derivatives on some rectangle R. If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, then $e^{\int f(x)dx}$ is an integrating factor of the differential equation $M dx + N dy = 0$ 5+5=10

10. a) Solve : $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

    b) Solve : $(1+2x^2)\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$,

    $-\frac{1}{2} < x < \infty$ 5+5=10

11. a) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x\sin(\log x), 0 < x < \infty$

    b) Use method of variation of parameter to solve

    $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$ 5+5=10

5. a) Find the eigen-values and corresponding eigen-vectors of the following matrix :

$$\begin{bmatrix} 6 & -12 & -5 \\ 1 & -7 & -5 \\ -2 & 12 & 9 \end{bmatrix}$$

   b) Prove that the eigenvalues of a real symmetric matrix are all real. 6+4

6. a) Verify Cayley-Hamilton theorem for A. Hence compute A^{-1}, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

   b) Solve the equation $x^8 + x^4 + 1 = 0$ using DeMoivre's theorem. 6+4

7. a) If $\tan(\theta + i\phi) = e^{i\alpha}$ show that $\theta = \left(n + \frac{1}{2}\right)\pi/2$ and

   $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$ (n is any integer).

   b) Expand $\sin^7\theta \mathrm{Cos}^2\theta$ in a series of sines of multiples of θ. 6+4